Reasoning about Interference (or lack thereof)

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My prejudices

- verification tackled by von Neumann and Turing
  - the search has been for ‘tractability’ [Jon03]
- the real payoff from formal methods is in design
  - code analysis can detect errors
  - unfortunately it does!
  - is it more productive to avoid their insertion?
  - formalism should support/check intuition
  - ‘posit and prove’
  - furthermore: a genie offers you a choice ...
- we understand complex systems from the top-down
  - i.e. via abstractions
  - ‘compositionality’ is a practical concern
- what if one is stuck with 10^n lines of legacy code?
  - top-down abstractions are useful in bottom-up analysis

Some lessons from VDM

VDM program development [Jon80]; VDM language description [BJ82]

- predicate restricted types (thanks to Lockwood Morris)
- relations (over Σ) are essential for post conditions ($\ldots$)
- 'real world' specifications don't fit on a line
    - keywords for structure
    - ... also for defining (**rd**/**wr**) 'frame'
- data abstraction/reification crucial in design
    - retrieve functions
    - 'adequacy'
- (avoid) 'implementation bias'
    - but there are understood cases where one can't
- cf. reservations about ghost (auxiliary) variables

Concurrency

- my main interest is in ‘shared variable’ concurrency
  - others have developed similar ideas for processes (communication-based) concurrency
  - interference: because construct ‘variables’ in π-calculus
- the real payoff from formal methods is in design
- concurrency
  - inherent in some top-level applications
  - for performance = distribute computation or data
- ‘compositionality’
  - is a practical concern
  - is difficult to achieve for concurrency

Issues in concurrency

look at issues (don't start with notation(s))

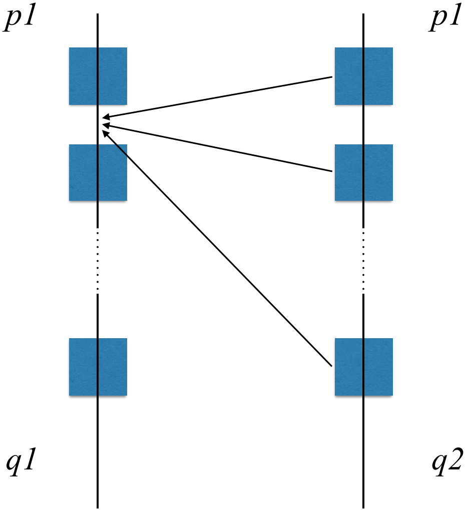
- interference
- separation
- ownership
- 'linearisability' (vs. splitting atoms)
- progress
- ...
- distributing data

Concurrency: pre Owicki

- Hoare (post *Axiomatic Basis*)
  - [Hoa69] — pick up [Hoa72] under "separation"
- interference (i.e. shared alphabets)
- Ashcroft [Ash75] (TR in 1973)
  - proof of "cross product" of control points
  - labour intensive!
- completely *post facto*
- non compositional
- arbitrary/fixed granularity assumption
  - assignments taken to be atomic
  - cf. so-called "Reynold's rule"

Susan Owicki [Owi75]

supervised by David Gries



Owicki/Gries [Owi75, OG76]

- interference (i.e. shared alphabets)
- separate sequential reasoning
- *post facto*: final ‘Einmischungsfrei’ PO
- non compositional
- arbitrary/fixed granularity assumption

Rely/Guarantee (R/G) idea is simple

face interference (in specifications and design process)

$$\underbrace{\overbrace{\sigma_0}^{pre} \quad \cdots \quad \overbrace{\sigma_i\ \sigma_{i+1}}^{rely} \quad \cdots \quad \underbrace{\sigma_j\ \sigma_{j+1}}_{guar} \quad \cdots \quad \sigma_f}_{post}$$

- assumptions $pre/rely$
- commitments $guar/post$

rely relations are an abstraction of interference to be tolerated
their expressive weakness might be a good thing!
'power' can beget intractability

One 5-tuple proof rule

many other possible rules

$$\boxed{Par\text{-}I}\ \frac{\begin{array}{l}\{P, R \vee G_2\}\ S_1\ \{G_1, Q_1\}\\ \{P, R \vee G_1\}\ S_2\ \{G_2, Q_2\}\end{array}}{\{P, R\}\ S_1 \mid\mid S_2\ \{G_1 \vee G_2, Q_1 \wedge Q_2 \wedge (R \vee G_1 \vee G_2)^*\}}$$

scope for variation in rules *much* larger (than in Hoare logic)
here: for composition (more compact than decomposition)
but, actually, less useful

Developments from R/G

- early
  - over 20 theses
  - zB Ketil Stoelen deals with progress
  - ...
  - Leonor Prensa Nieto formalise soundness proof in Isabelle
- ...
- recent
  - RGSep [Vaf07]
  - Bornat (twice) on Simpson's '4-slot' algorithm
  - (Concurrent) Kleene Algebras
  - Armstrong (2016) looks at soundness via Kleene Algebras

R/G rethought [JHC15, HJC14]

"pulling apart" R/G

Original rely/guarantee (R/G) can be presented as 5-tuples

But instead of
$\{P, R\}\ S\ \{G, Q\}$

We now follow the 'refinement calculus':
$x{:}\ \left[P, Q\right]$
$x{:}\ \left[Q\right]$
$=\ /\ \sqsubseteq$

and wrap **rely**/**guar** around *any* statement
rely $R \bullet c$
guar $G \bullet c$

(Some) Laws of the new algebraic R/G

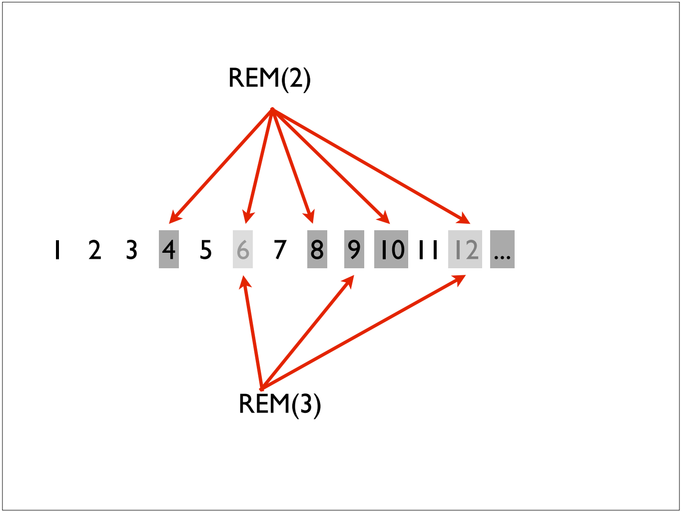
Nested-G: $(\textbf{guar}\ g_1 \bullet (\textbf{guar}\ g_2 \bullet c)) = (\textbf{guar}\ g_1 \wedge g_2 \bullet c)$

Intro-G: $c \sqsubseteq (\textbf{guar}\ g \bullet c)$

Trading-G-Q: $(\textbf{guar}\ g \bullet [g^* \wedge q]) = (\textbf{guar}\ g \bullet [q])$

Intro-multi-Par: $\wedge_i [q_i] \sqsubseteq \|_i\ (\textbf{guar}\ gr \bullet (\textbf{rely}\ gr \bullet [q_i]))$
(asymmetric version below)

(Parallel) Sieve of Eratosthenes



Refinement calculus style development

Set s might initially contain all natural numbers up to some n
C is the set of all composite numbers

$$[s' = s - C] = [s' \subseteq s \wedge s - s' \subseteq C \wedge s' \cap C = \{\,\}]$$

$\sqsubseteq$ by Intro-G

$$\textbf{guar}\; s' \subseteq s \wedge s - s' \subseteq C \bullet [s' \subseteq s \wedge s - s' \subseteq C \wedge s' \cap C = \{\,\}]$$

$=$ by Trading-G-Q ($s - s' \subseteq C$ is transitive)

$$\textbf{guar}\; s' \subseteq s \wedge s - s' \subseteq C \bullet [s' \cap C = \{\,\}]$$

$\sqsubseteq$ by Intro-multi-Par

$$\textbf{guar}\; s' \subseteq s \wedge s - s' \subseteq C \bullet \\ (\|_i \;\textbf{guar}\; s' \subseteq s \bullet \textbf{rely}\; s' \subseteq s \bullet [s' \cap c_i = \{\,\}])$$

$=$ Nested-G

$$\textbf{guar}\; s - s' \subseteq C \wedge s' \subseteq s \bullet (\|_i \;\textbf{rely}\; s' \subseteq s \bullet [s' \cap c_i = \{\,\}])$$

R/G observations

- asymmetric rely/guarantee conditions are important:
  $[q_1 \wedge q_2] \sqsubseteq (\textbf{guar}\ g_1 \bullet (\textbf{rely}\ g_2 \bullet [q_1])) \parallel (\textbf{guar}\ g_2 \bullet (\textbf{rely}\ g_1 \bullet [q_2]))$
- nice bonus of new style: **guar-inv** $g \bullet c$
- apposite representations often key to avoiding locking
  - FINDP
  - QREL — (spotted in design of CLEANUP)
  - SIEVE
  - 4-SLOT
  - only fully realised in [Jon07]
- (yet more) 'abstract R/G' (Ian Hayes looking at **guar** $c \bullet c'$)
  - cf. 'phasing', but
  - remember 'expressive weakness' point!

Another issue: ‘separation’

- ‘separation’ = ‘non-interference’
  - return to this later
- see what (data) abstraction can do for interference
  - retaining my top-down prejudice
- ordinary (‘scoped’) variables assumed to be separate
  - leave aside ‘by location’ parameter passing!
- towards a different view
  - ‘heap’ variables as representations of scoped variables?
- this exercise in the same spirit as ‘taking apart’ R/G

Separation Logic (SL)

refresher!

- basic idea is, again, simple
  - to prove things about $S_1 \parallel S_2$
  - would like to conjoin their pre/post conditions
- history
  - parallelism with ‘scoped’ variables — [Hoa72]
  - mentioned by Peter O’Hearn at Tony’s 2009 event
  - ‘Separation Logic’ for ‘heap’ variables — [Rey02]
  - Concurrent Separation Logic — Peter O’Hearn [O’H07]
- ‘heap’ variables can’t be handled by ‘alphabets’
  - SL designed for this case
- origin: bottom-up code analysis
  - heap variables
  - “probably avoid SL for ‘scoped’ variables!”

Two key SL proof rules

'Separating conjunction' – $P * Q$ (only if P and Q are separate)

$$\boxed{\text{SL-concurrency}}\ \frac{\begin{array}{l}\{P_1\}\ s_1\ \{Q_1\}\\ \{P_2\}\ s_2\ \{Q_2\}\end{array}}{\{P_1 * P_2\}\ s_1 \mid\mid s_2\ \{Q_1 * Q_2\}}$$

Frame rule

$$\boxed{\text{SL-frame}}\ \frac{\{P\}\ s\ \{Q\}}{\{P * R\}\ s\ \{Q * R\}}$$

‘Separation as an abstraction’

[JY15]

two examples:

- Reynold’s simple sequential (in-place) list reversal
- a concurrent merge sort

Example 1: list reversal example (Reynolds)

John started . . .

The following *program* performs an in-place reversal of a list:

$j := \textbf{nil};\ \textbf{while}\ i \neq \textbf{nil}\ \textbf{do}$
$\quad (k := [i + 1];\ [i + 1] := j;\ j := i;\ i := k).$

(Here the notation $[e]$ denotes the contents of the storage at address e.)

He then derives a post condition using $\exists \alpha, \beta \cdot list(\alpha, i) * list(\beta, j)$

Re-do the example with
‘Separation as an abstraction’

$$r, s \colon \left[r' = rev(s) \right]$$

$$rev : X^* \to X^*$$
$$rev(s) \quad \triangleq \quad \cdots$$

s and r are *assumed* to be distinct (‘scoped’) variables
that they are separate is a (useful and) natural abstraction

first step of design

$$\Sigma_0 = X^* \times X^*$$

it is straightforward to 'posit & prove':

$r \leftarrow [\,]$;
while $s \neq [\,]$ **do**
  $STEP_0$
  $\{rev(s') \frown r' = rev(s) \frown r \wedge \textbf{len}\, s' < \textbf{len}\, s\}$
od

$$STEP_0 \;\; r, s\colon \left[s \neq \{\,\},\; r' = [\textbf{hd}\, s] \frown r \wedge s' = \textbf{tl}\, s\right]$$

We have finished thinking about reversing sequences!
We now think about data (representation)
NB: s and r are still assumed to be distinct variables

($Heap$ but) $Srep$ is a useful abstraction

$$Heap = Ptr \xrightarrow{m} (X \times [Ptr])$$

Inductive definition of $Srep \subseteq Heap$

$$\{\,\} \in Srep$$

$$sr \in Srep \land p \in Ptr \land p \notin \mathbf{dom}\, sr \;\Rightarrow\; (\{p \mapsto (v, start(sr))\} \cup sr) \in Srep$$

$$start(\{\,\}) = \mathbf{nil}$$

$$start(\{p \mapsto (v, start(sr))\} \cup sr) = p$$

Could develop a theory of $Srep$ (e.g. Isabelle)

Reify $(X^* \times X^*)$ as Σ_1

$$\Sigma_1 = (Srep \times Srep)$$

where

$$inv\text{-}\Sigma_1((sr, rr)) \quad \triangleq \quad sep(sr, rr)$$

$$sep : Srep \times Srep \to \mathbb{B}$$

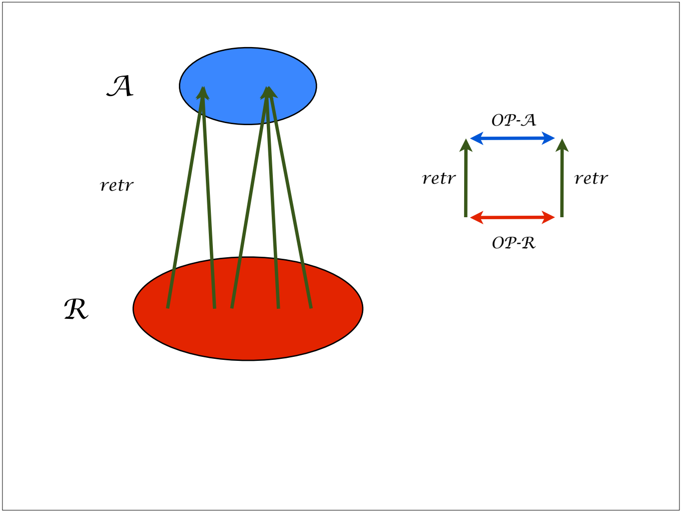
$$sep(sr, rr) \quad \triangleq \quad \mathbf{dom}\, sr \cap \mathbf{dom}\, rr = \{\,\}$$

$STEP$ on Σ_1

$$
STEP_1 \quad rr, sr: \left[\begin{array}{l}
sr \neq \{\,\}, \\
\textbf{let } p = start(sr) \textbf{ in} \\
\quad sr' = \{p\} \triangleleft sr \wedge \\
\quad rr' = rr \cup \{p \mapsto (sr(p)_1, start(rr))\}
\end{array}\right]
$$

Lemma 1 $STEP_1$ preserves $inv\text{-}\Sigma_1$

Data reification (homomorphic rule)



Data reification proof — standard (VDM) rule

$$retr_0 : \Sigma_1 \to \Sigma_0$$

$$retr_0((sr, rr)) \quad \triangleq \quad (gather(sr), gather(rr))$$

$$gather: Srep \to X^*$$
$$gather(\{\ \}) = [\,]$$
$$gather(\{p \mapsto (v, start(sr))\} \cup sr) = [v] \curvearrowright gather(sr)$$

Lemma 2 ('Adequacy') There is a Σ_1 representation of any Σ_0

Lemma 3 ('Commutativity') $STEP_1$ models (under $retr_0$) the abstract $STEP_0$

Reification to a single $Heap$

$$\Sigma_2 = (Heap \times Ptr \times Ptr)$$

where

$$\begin{array}{l} inv\text{-}\Sigma_2((hp, i, j)) \quad \triangleq \\ \qquad \exists sr, rr \in Srep \cdot sr \cup rr \subseteq hp \ \wedge \ i = start(sr) \wedge j = start(rr) \end{array}$$

- another exercise in data reification
- it is mandatory that sep holds between the two sub-heaps because their union is used in $(sr \cup rr) \subseteq hp$
- NB $\subseteq$ admits the possibility of other information in the heap

Relating Σ_2/Σ_1

$$retr_1 : \Sigma_2 \to \Sigma_1$$

$$retr_1((hp, i, j)) \quad \triangleq \quad (trace(hp, i) \lhd hp, trace(hp, j) \lhd hp)$$

$$trace : Heap \times Ptr \to Ptr\text{-}\mathbf{set}$$

$$trace(hp, p) \quad \triangleq \quad \begin{array}{l} \mathbf{if}\ p = \mathbf{nil} \\ \mathbf{then}\ \{\,\} \\ \mathbf{else}\ \{p\} \cup trace(hp, hp(p)_2) \end{array}$$

Lemma 4 $trace$ from $start(sr)$ characterises sr

Lemma 5 ('Adequacy') of Σ_2 wrt $\Sigma_1/retr_1$

Theorem 1 ('Commutativity') $STEP_2$ models (under $retr_1$) $STEP_1$

Comments on Example 1

- *separation is an abstraction*
  - wot, no Separation Logic?
  - representation shown to preserve the abstraction
  - standard reification process
- wot, no R/G?
  - no concurrency
  - therefore, no interference
- layered design
  - (only) first step is concerned with list reversal
  - second is (only) about data representation
- (C++) code in the paper

Example 2: *concurrent* merge sort

SKIP!

$$is\text{-}sort : X^* \times X^* \to \mathbb{B}$$

$$is\text{-}sort(s, s') \quad \triangleq \quad ordered(s') \wedge permutes(s', s)$$

$$ordered : X^* \to \mathbb{B}$$

$$ordered(s) \quad \triangleq \quad \cdots$$

$$permutes : X^* \times X^* \to \mathbb{B}$$

$$permutes(s, s') \quad \triangleq \quad \cdots$$

Because concurrent processes are used, employ *Intro-par*

What has been achieved?

- reason about *separation as an abstraction*
  - only standard (long-established) notions
- (like all reification steps) argue properties preserved
- key sorting ideas proved on the abstraction
  - only need to show the implementation mirrors steps
  - echoes Wirth: *Algorithms + Data = Programming*
- minimal use of R/G, mainly abstraction!
- this is *not* an argument against SL
  - … a nice definition of $Srep$ uses separating conjunction (*)
  - as with ‘pulling apart’ R/G, get to issue (of separation)

A (non-specialist's) view of SL

- *basic idea* works well for 'disjoint concurrency'
  - e.g. parallel merge sort
- (most papers) limit to 'partial correctness'
- (too?) many extensions
  - magic wand (fits algebraic view)
  - fractional permissions Boyland
  - Concurrent Abstract Predicates [DYDG$^+$10]
  - *Next 700 Separation Logics* [Par10]
- conceptual framework
  - monoids
  - Abstract Separation Logic [COY07]
  - Views [DYBG$^+$13]
- is it better to have everything under one (conceptual) roof?
  - vs. (?)
  - 'natural abstractions'

Relating interference/separation

compare with RGSep [Vaf07]

A key (abstract) R/G law

$[q_1 \wedge q_2] \sqsubseteq (\textbf{guar}\ g_1 \bullet (\textbf{rely}\ g_2 \bullet [q_1])) \parallel (\textbf{guar}\ g_2 \bullet (\textbf{rely}\ g_1 \bullet [q_2]))$

... covers complete or partial separation

Conclusions

- don't take position:
  "my notation (aka hammer) solves every problem"
- beware the siren the call of 'universality'
- but 'abstraction' is a/the key to understanding
- start with the issues
  - interference
  - separation
  - ownership
  - progress
  - 'linearisability' (vs. splitting atoms)
  - ...
  - distributing data

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