1 Exercise Problems

Problem 1. The weather in Vienna is described by the following stochastic process: (i) if it is sunny today, then it is sunny tomorrow with probability 0.7 and rainy tomorrow with probability 0.3; and (ii) if it is rainy today, then it is sunny tomorrow with probability 0.5 and rainy tomorrow with probability 0.5. Show the stochastic process as a finite-state Markov chain. Calculate the average frequency of sunny and rainy days in Vienna.

Problem 2. A gambler walks into a Casino. He is happy if he has $\$4$ and leaves the Casino. To play one round, the gambler pays $\$1$: if he wins he gets $\$2$ back (i.e., total profit of $\$1$), otherwise he looses the amount to play.

1. Assume that he wins and looses in every round with probability $\frac{1}{2}$. Suppose he enters the Casino with $\$2$ (and leaves the Casino either when he has no money or $\$4$). Show the Markov chain and calculate the probability that he leaves the Casino with $\$4$.
2. Repeat the above item when he enters the Casino with $\$3$.
3. Repeat item(1) when the probability to win in every round is $1/3$, and to loose is $2/3$.

Problem 3. In this problem we consider how the almost-sure winning region in MDPs can change due to addition of edges. Consider the examples shown in Figure 1 where the right-hand part is the almost-sure winning region. Describe if the following addition of edges can change the almost-sure winning region: if the region cannot change, explain why, and if the region can change illustrate with examples. (1) Case 1: Addition of a probabilistic edge from the non-almost-sure winning region to the almost-sure winning region; (2) Case 2: Addition of a player 1 edge from the almost-sure winning region to the non-almost-sure winning region; (3) Case 3: Addition of a probabilistic edge from the non-almost-sure winning region to the non-almost-sure winning region; and (4) Case 4: Addition of a player 1 edge from the non-almost-sure winning region to the non-almost-sure winning region.

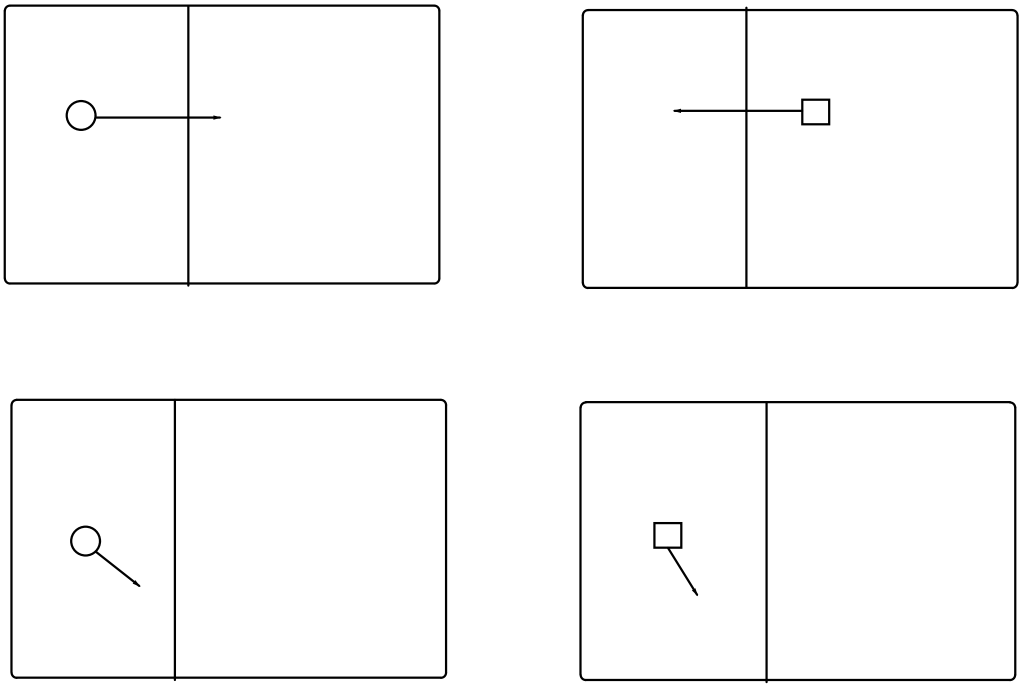

Fig. 1. Example