On Compiling CNFs into Structured Deterministic DNNFs

Friedrich Slivovsky

joint work with Simone Bova, Florent Capelli, and Stefan Mengel

Model Counting (#SAT)

Instance: A propositional formula F in CNF

Problem: Count the satisfying assignments of F

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#P-complete for HORN, monotone 2CNF

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#P-complete for HORN, monotone 2CNF

structural restrictions often yield tractability

Previous talk:
exact model counters implicitly compile CNFs into **decision** DNNFs

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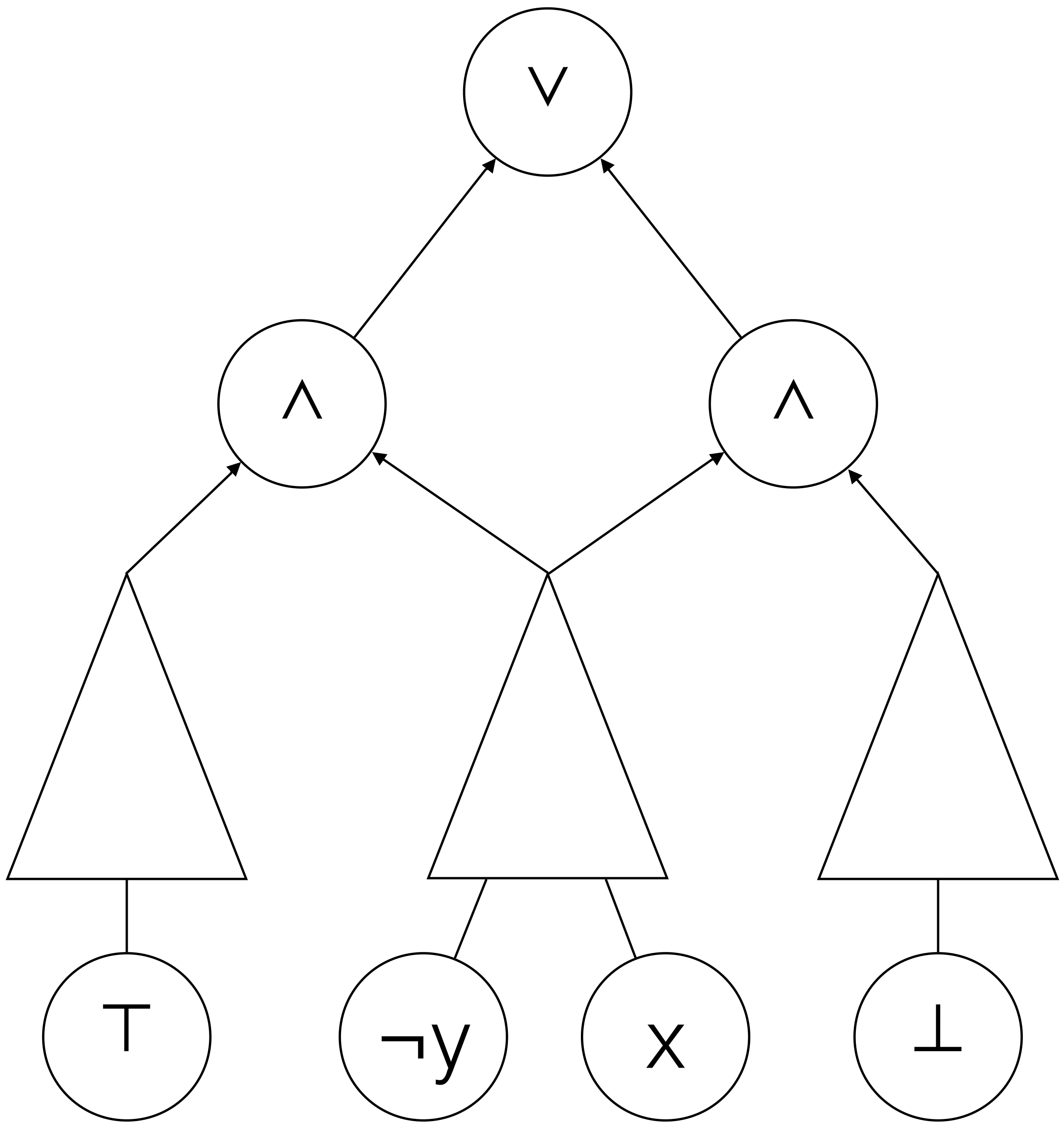
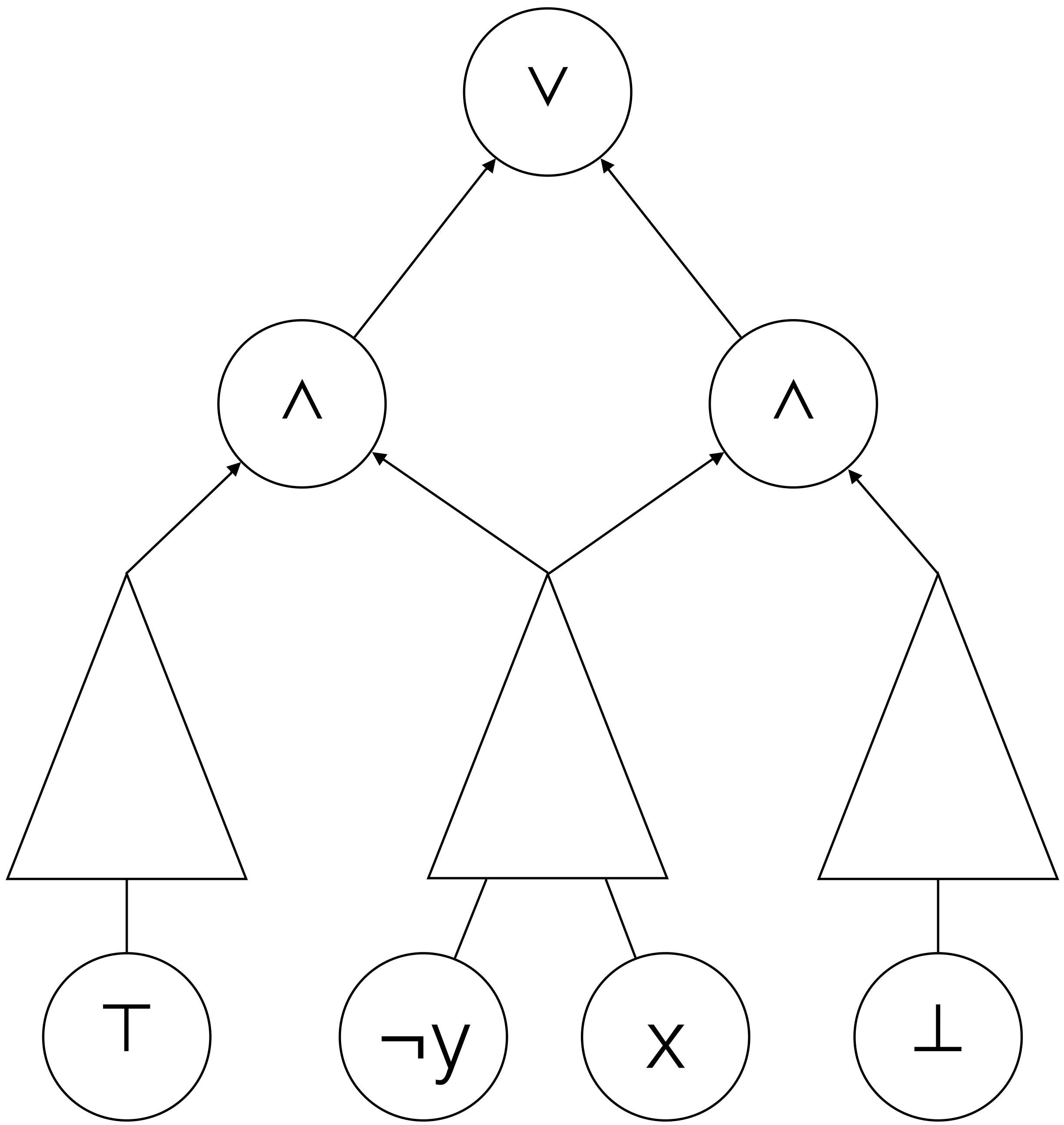
This talk:
compilation of CNFs into **structured deterministic** DNNFs based on new model counting algorithms

deterministic DNNF



deterministic DNNF

decomposable



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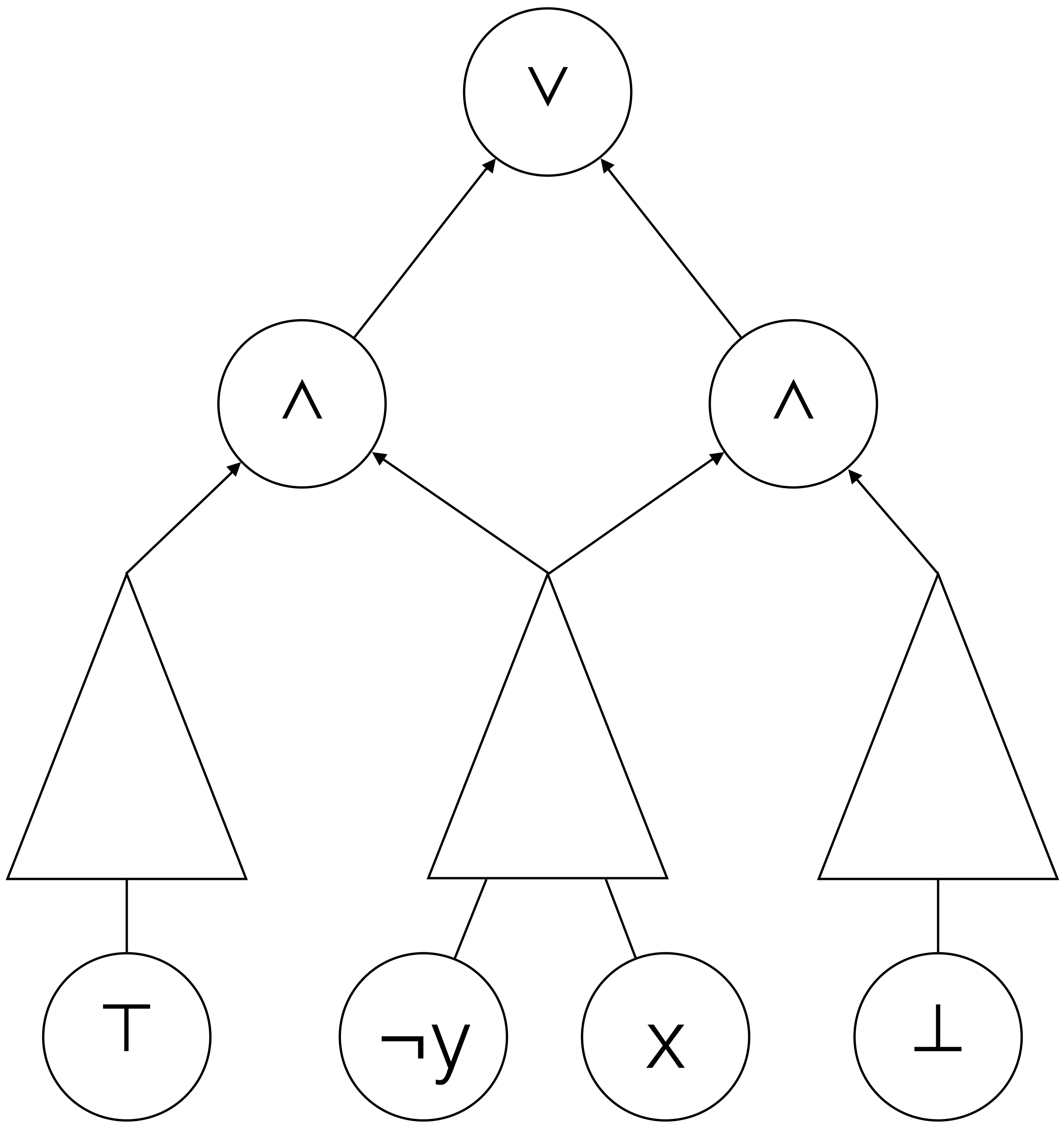

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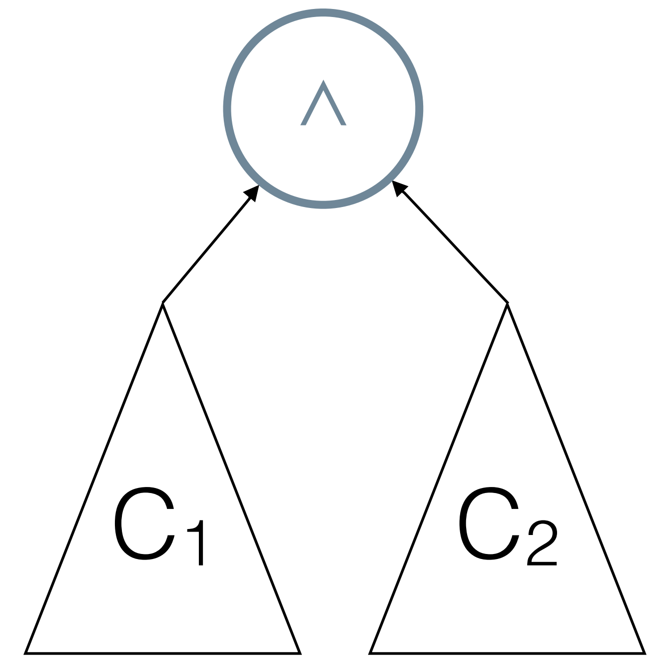

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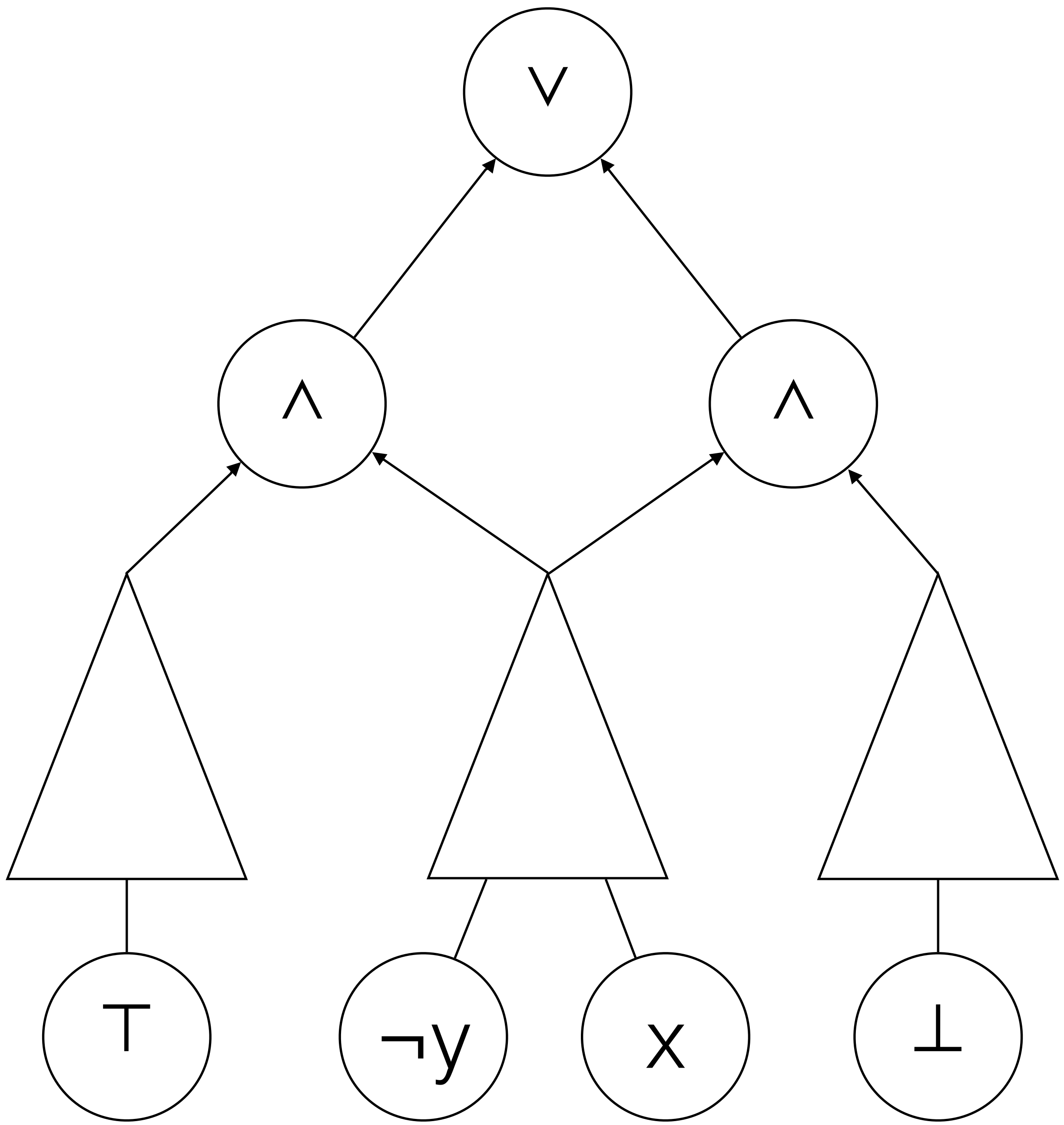

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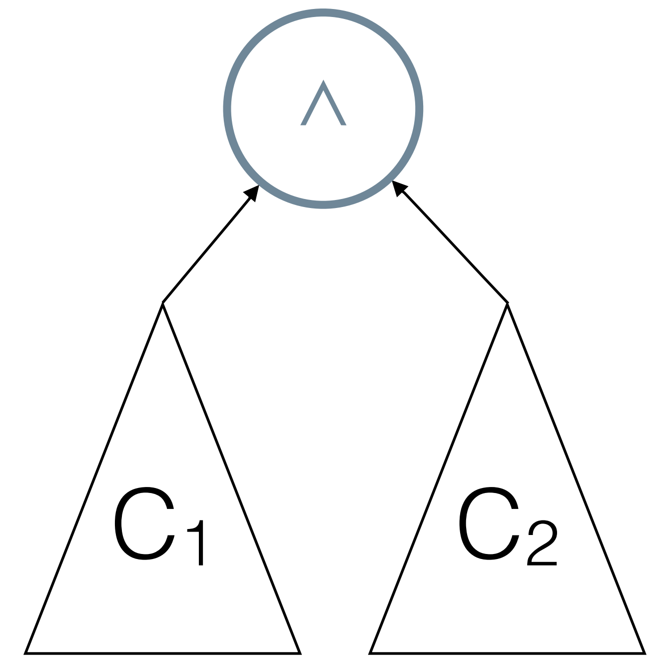
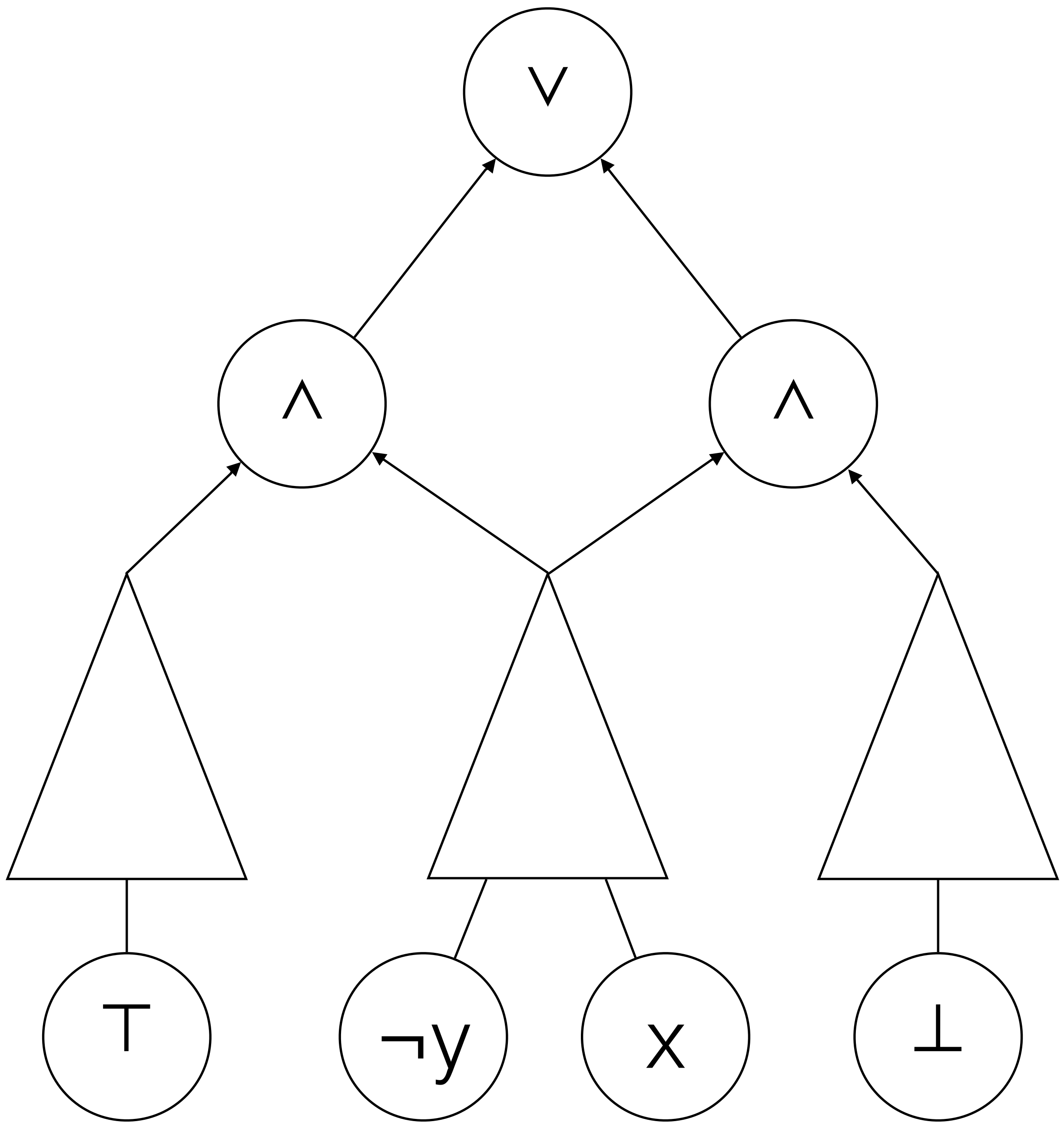


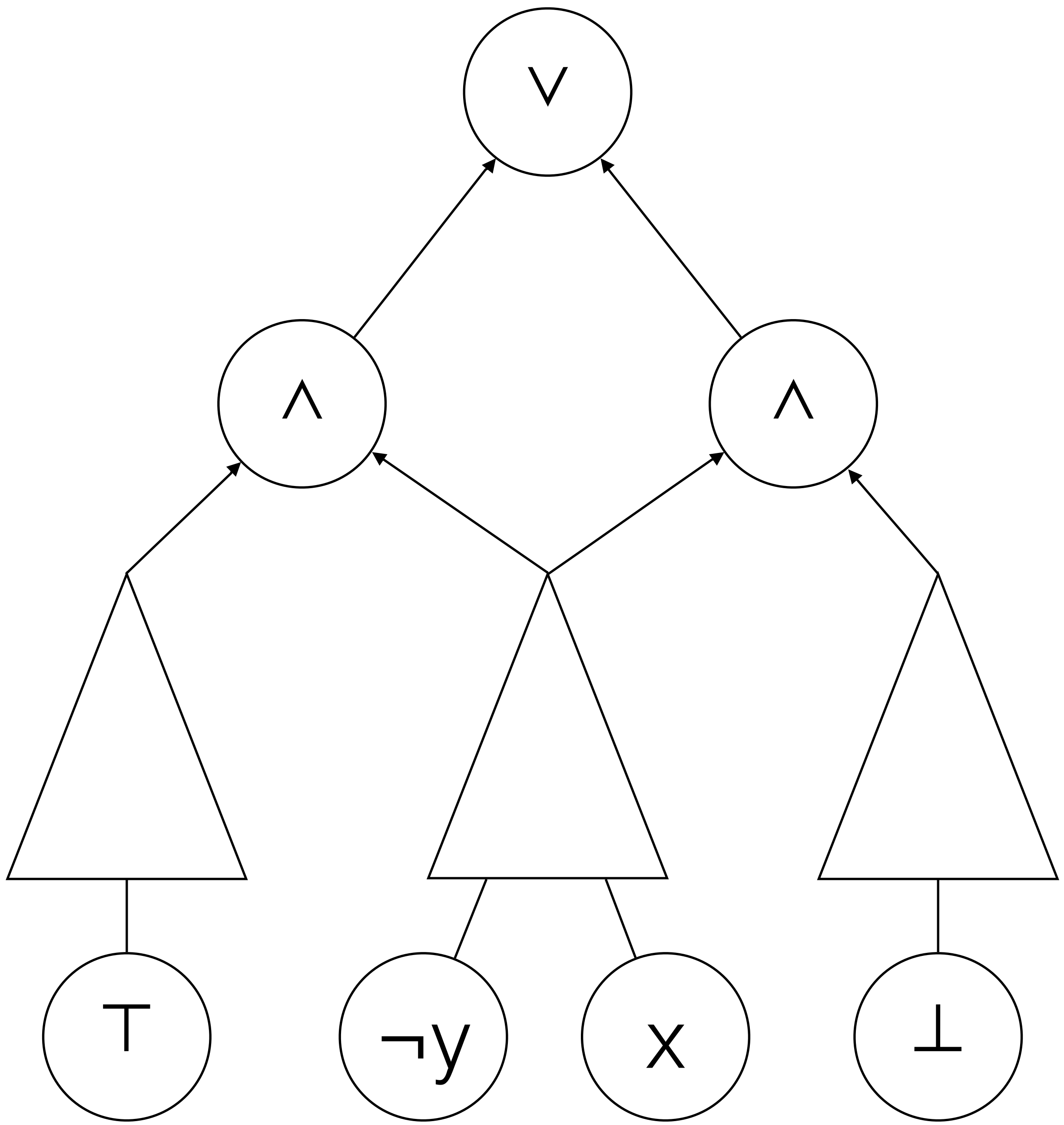
$var(C_1) \cap var(C_2) = \varnothing$

deterministic DNNF

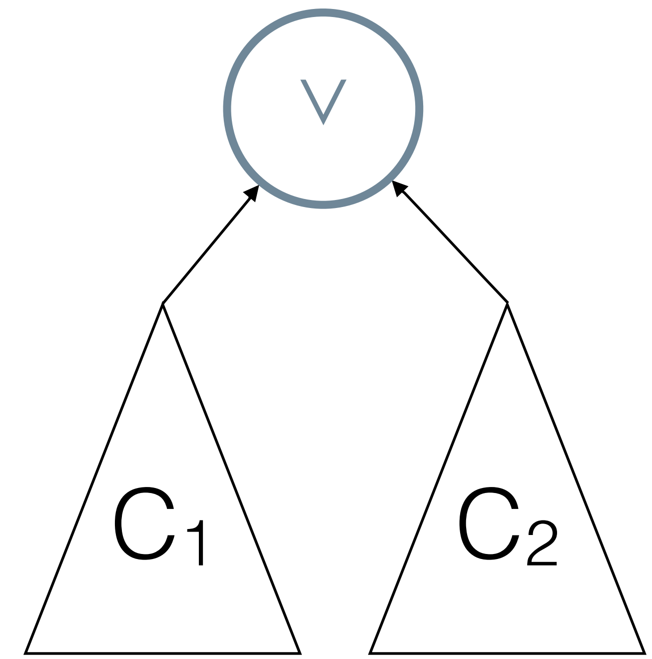


deterministic

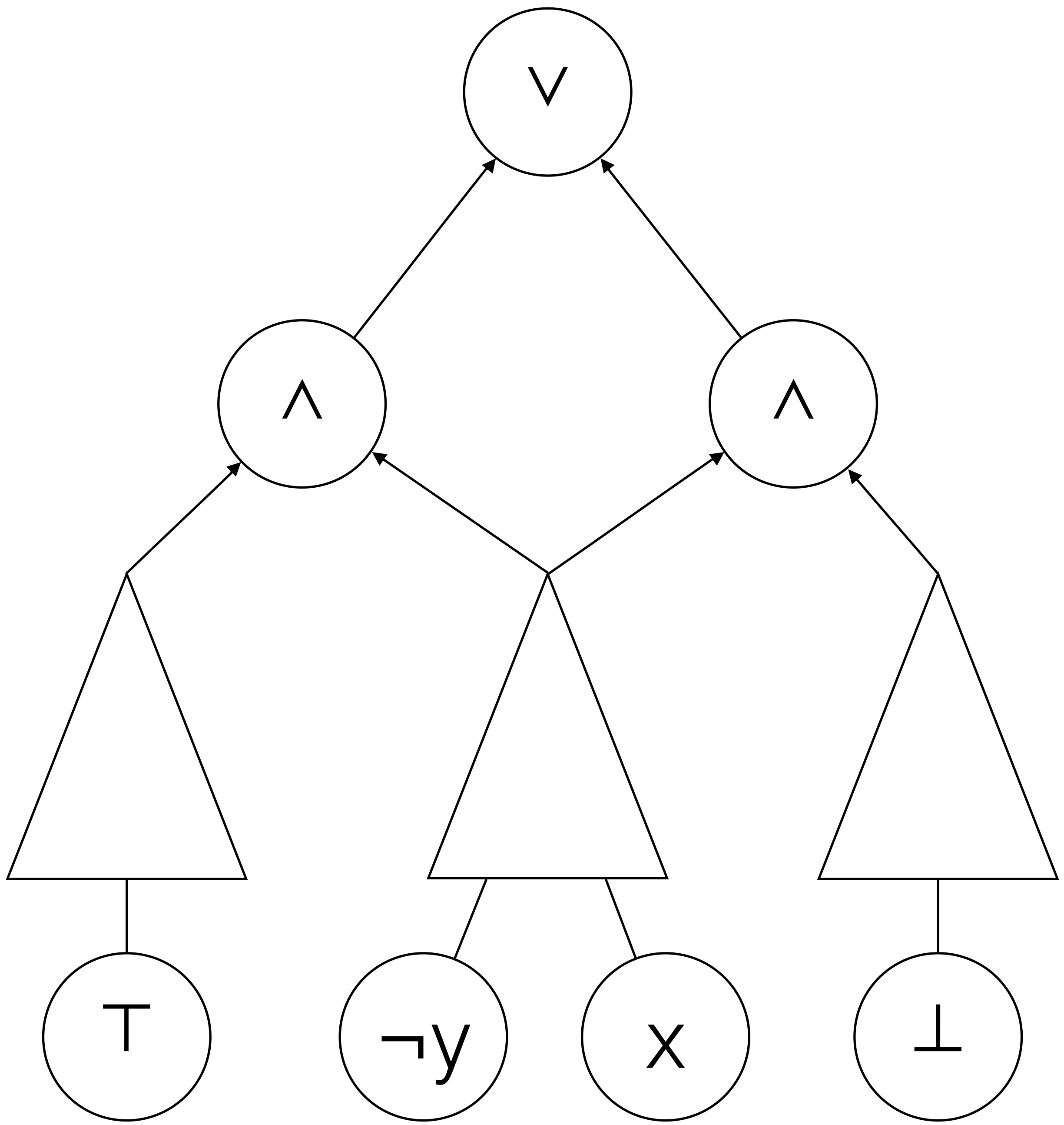
deterministic DNNF



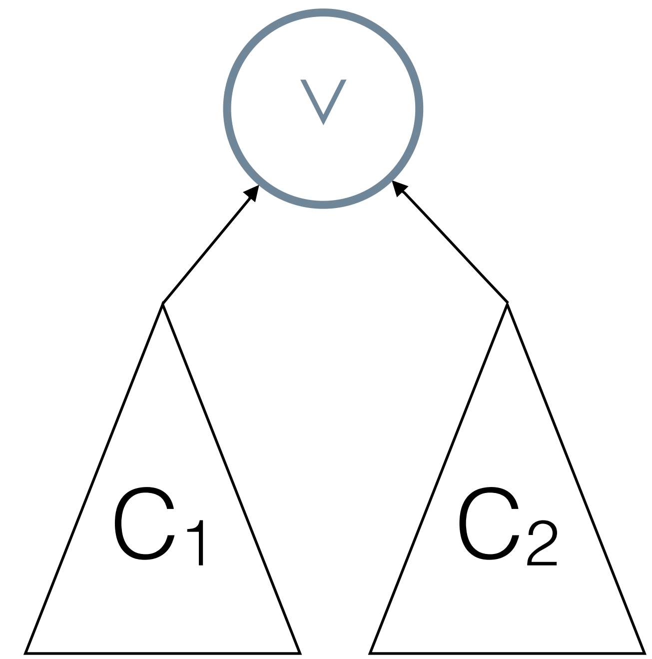
deterministic



deterministic DNNF



deterministic



$\text{models}(C_1) \cap \text{models}(C_2) = \varnothing$

CNFs are not **polysize compilable** into DNNFs

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unless PH collapses
(Selman & Kautz 1996)

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this can proved **unconditionally**
(Bova, Capelli, Mengel, S. 2014)

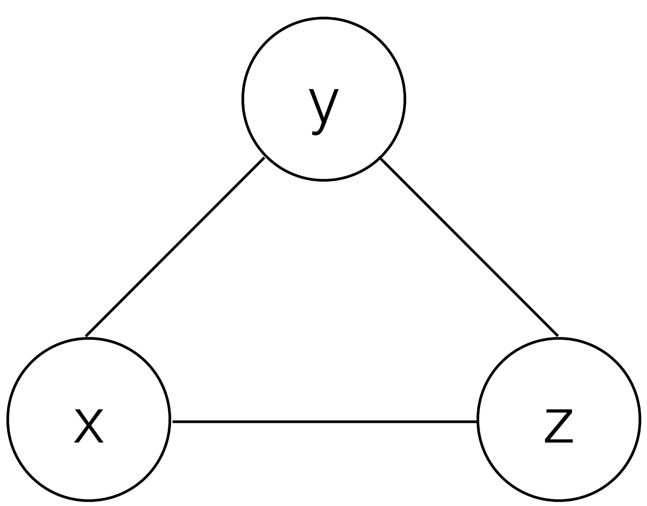
Structural Parameters

$(x \vee \neg y \vee z) \wedge (\neg x \vee \neg z) \wedge (y \vee z)$

Structural Parameters

$(x \lor \neg y \lor z) \land (\neg x \lor \neg z) \land (y \lor z)$

primal graph



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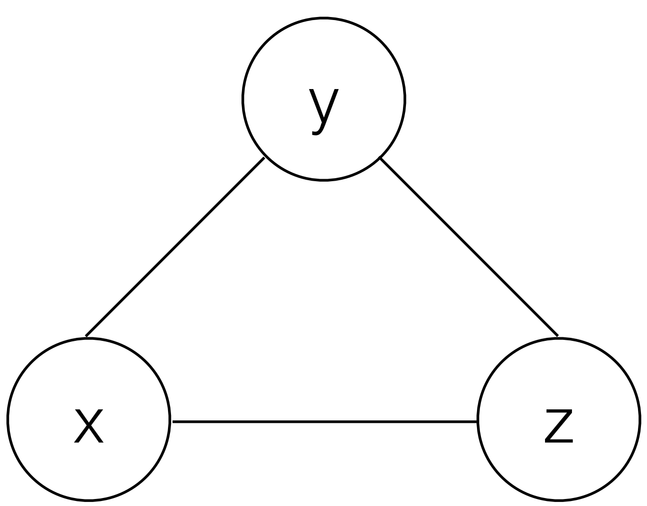
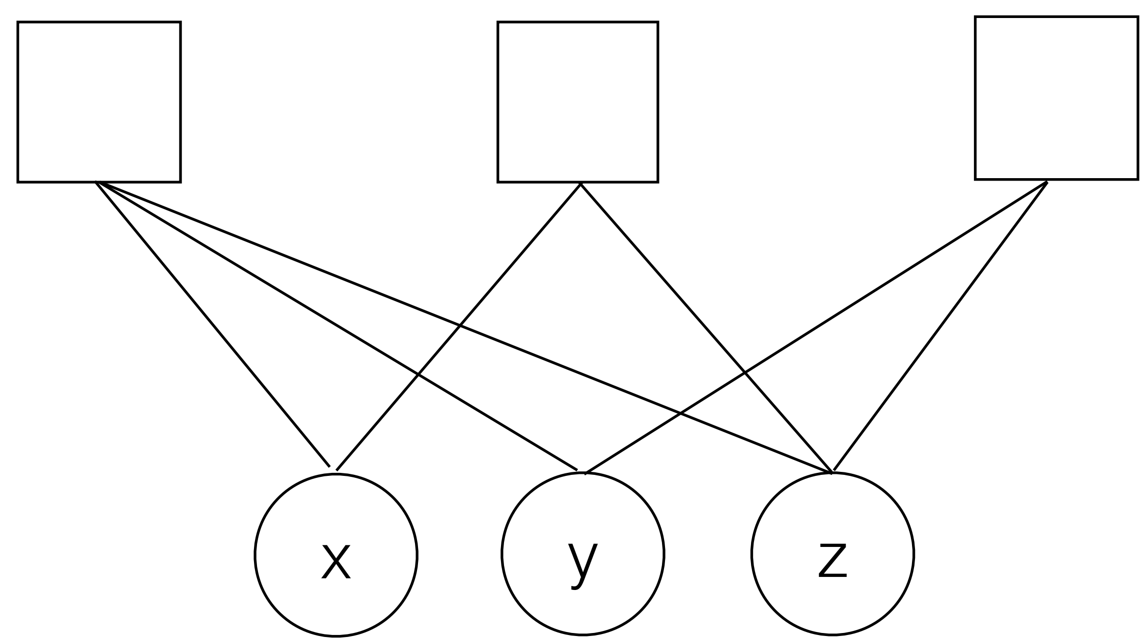


incidence graph



incidence treewidth

primal treewidth

incidence treewidth

primal treewidth

decision

$2^k n$

incidence treewidth

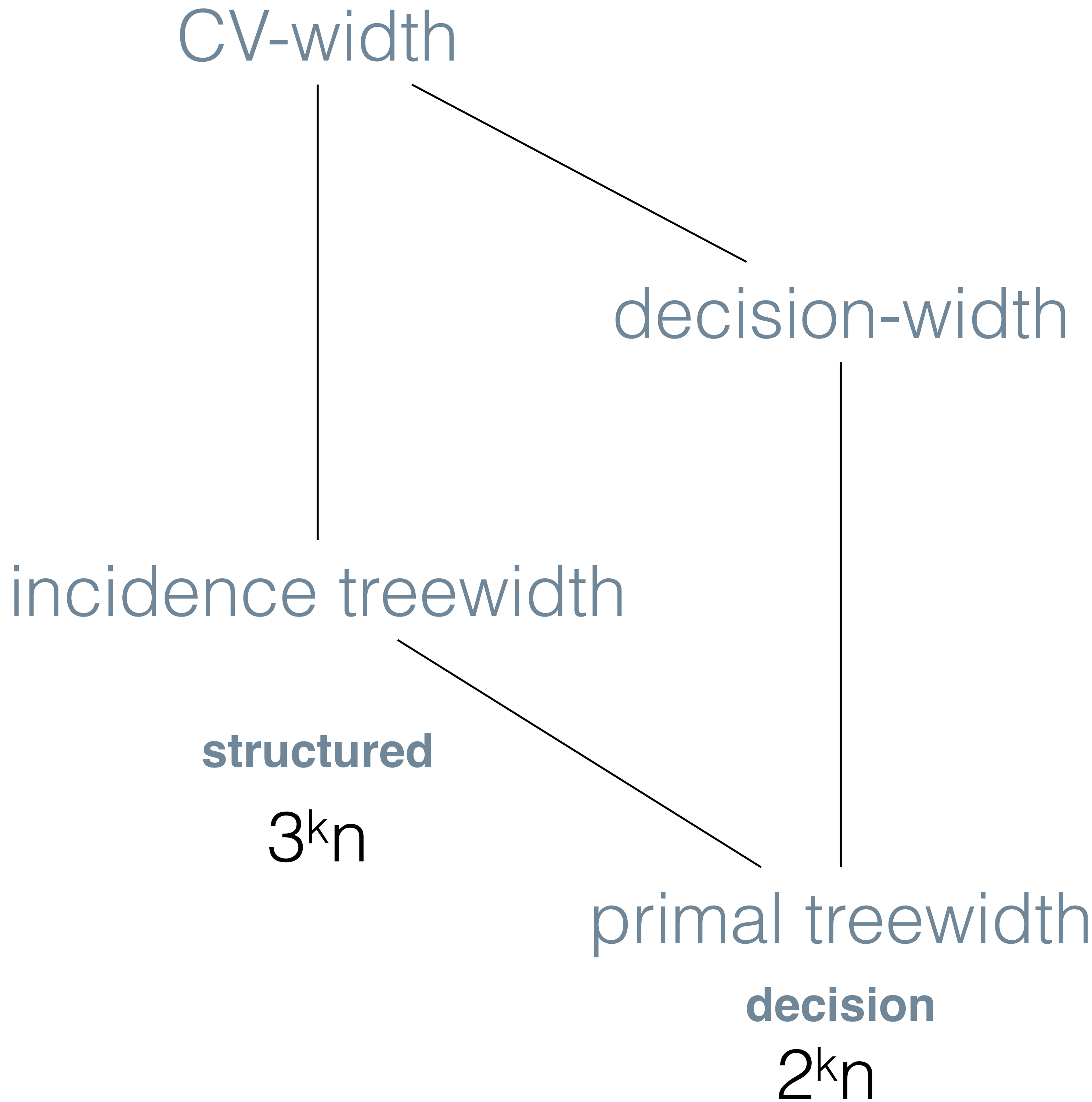
structured

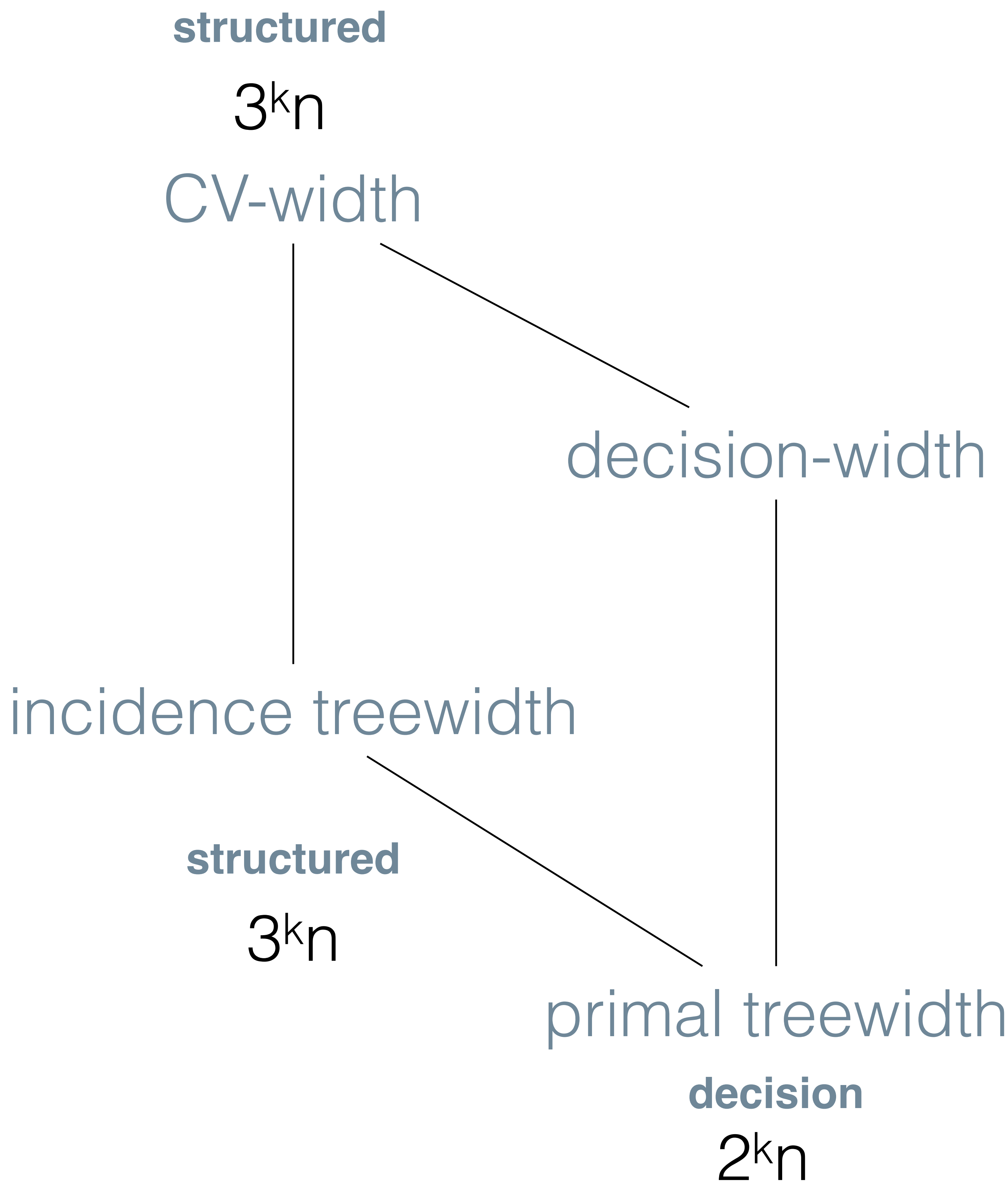
$3^k n$

primal treewidth

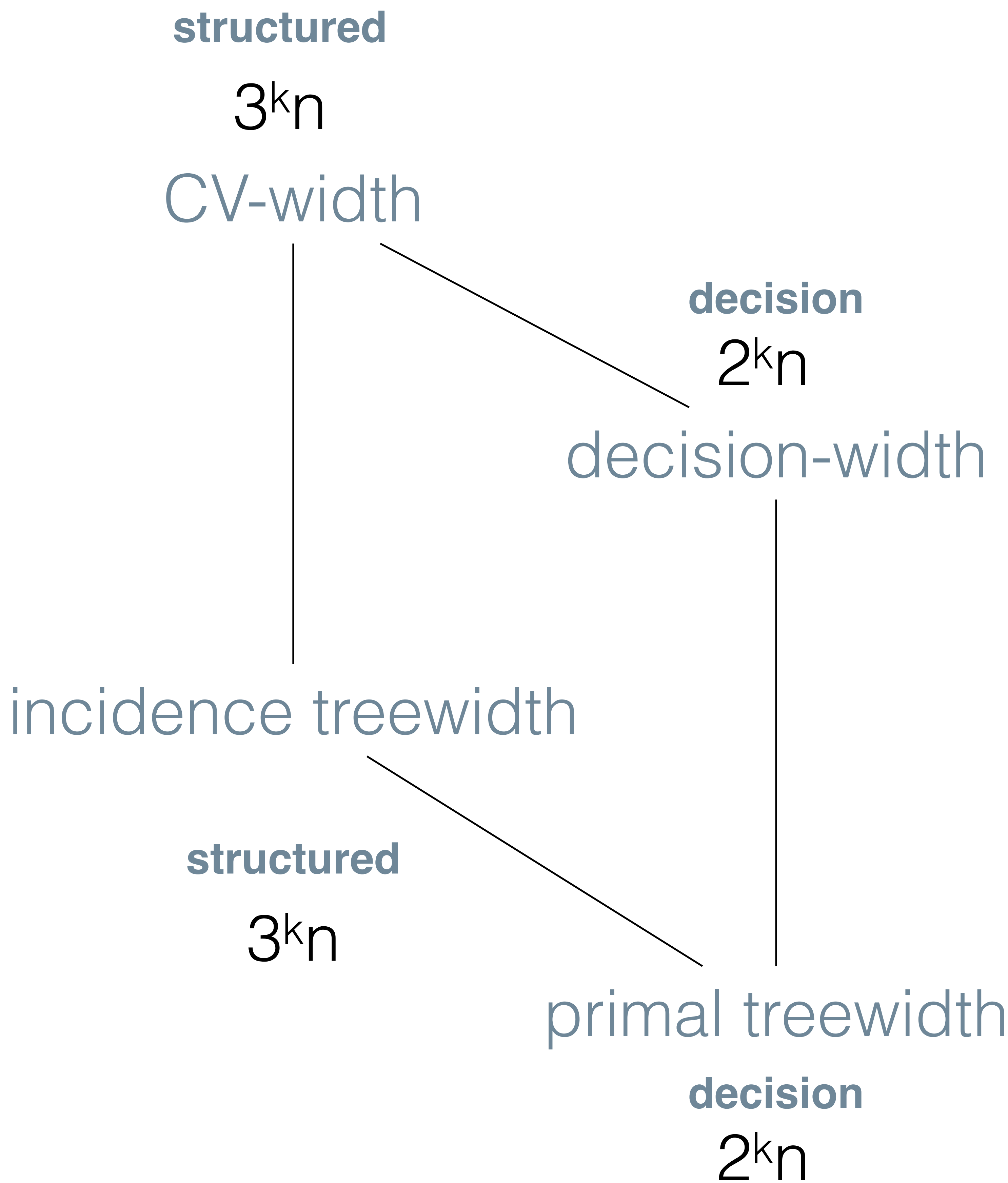
decision

$2^k n$

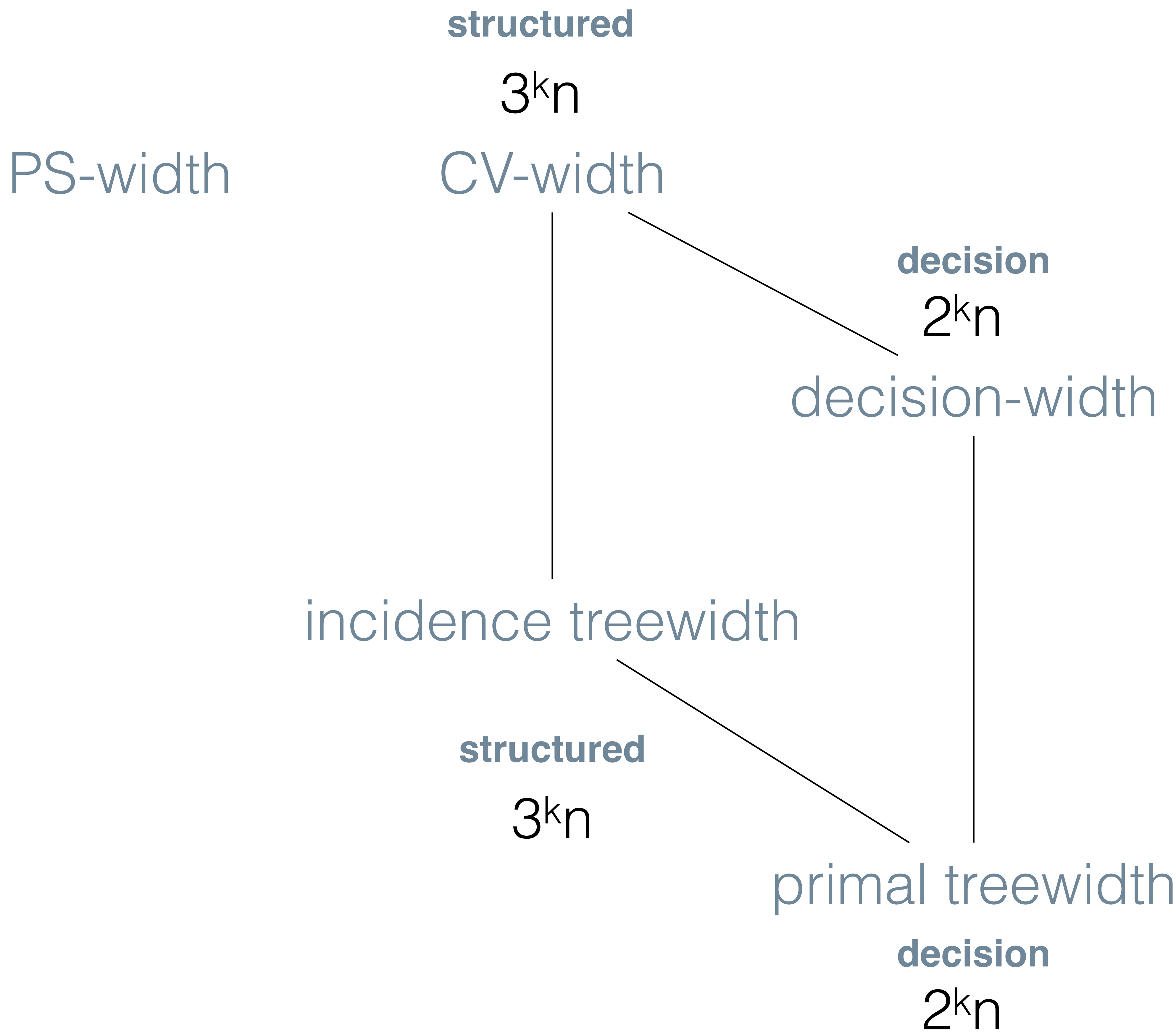

CV-width
decision-width
incidence treewidth
structured
3^k n
primal treewidth
decision
2^k n

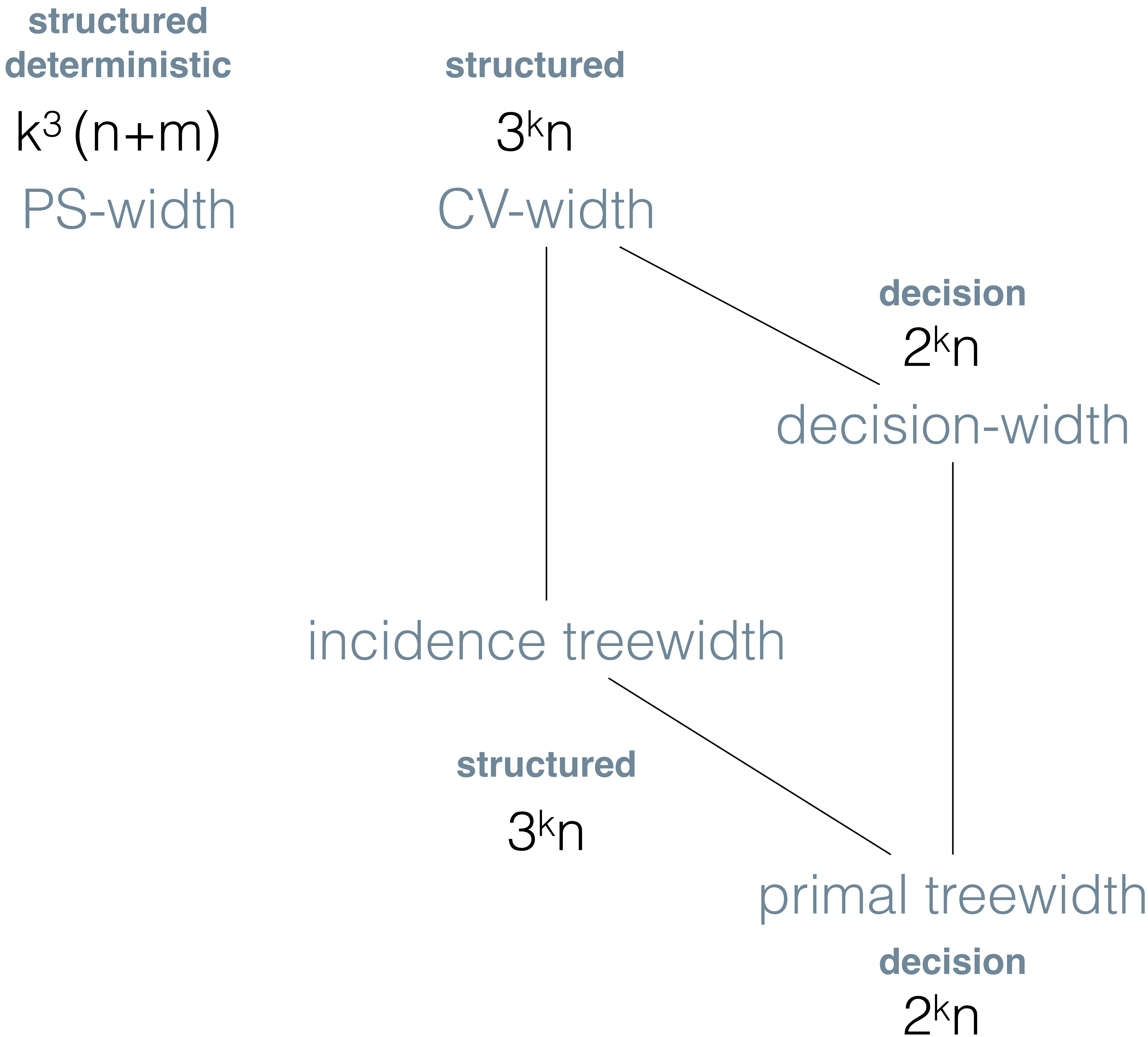


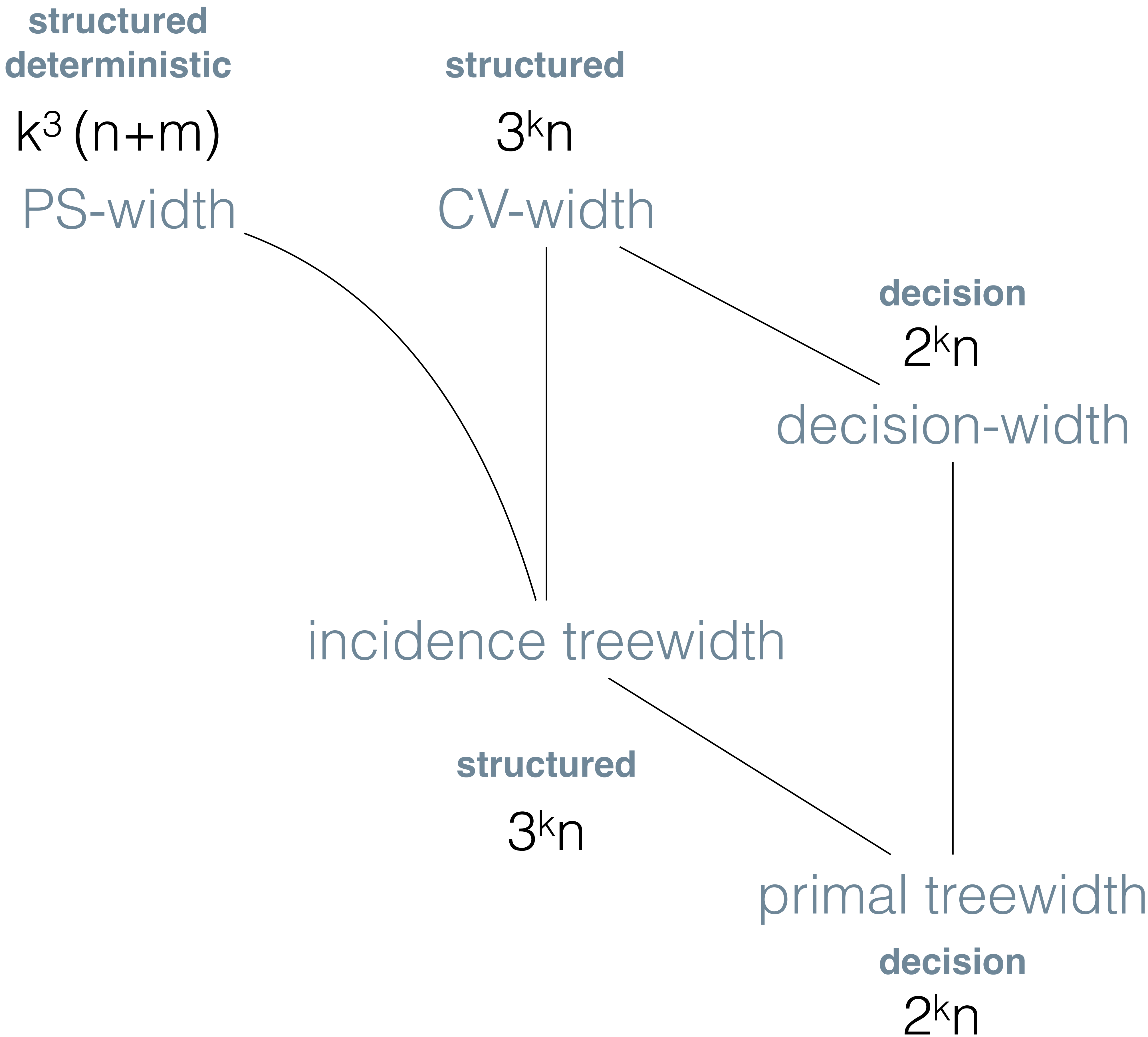
structured
3^k n
CV-width
decision-width
incidence treewidth
structured
3^k n
primal treewidth
decision
2^k n



structured
3kn
CV-width
decision
2kn
decision-width
incidence treewidth
structured
3kn
primal treewidth
decision
2kn


structured
3ᵏn
PS-width
CV-width
decision
2ᵏn
decision-width
incidence treewidth
structured
3ᵏn
primal treewidth
decision
2ᵏn


structured deterministic
k3 (n+m)
PS-width
structured
3kn
CV-width
decision
2kn
decision-width
incidence treewidth
structured
3kn
primal treewidth
decision
2kn


structured
deterministic
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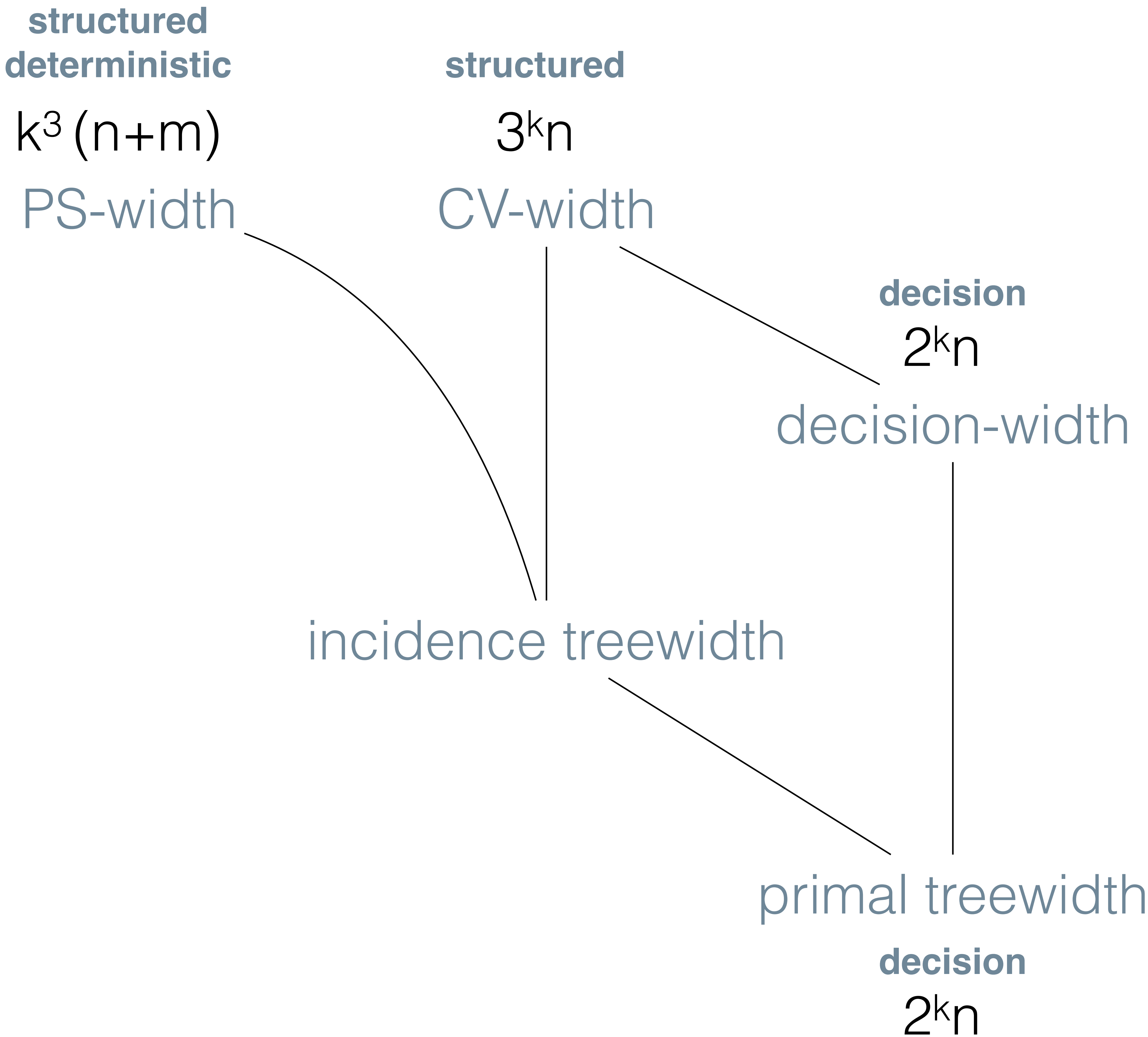
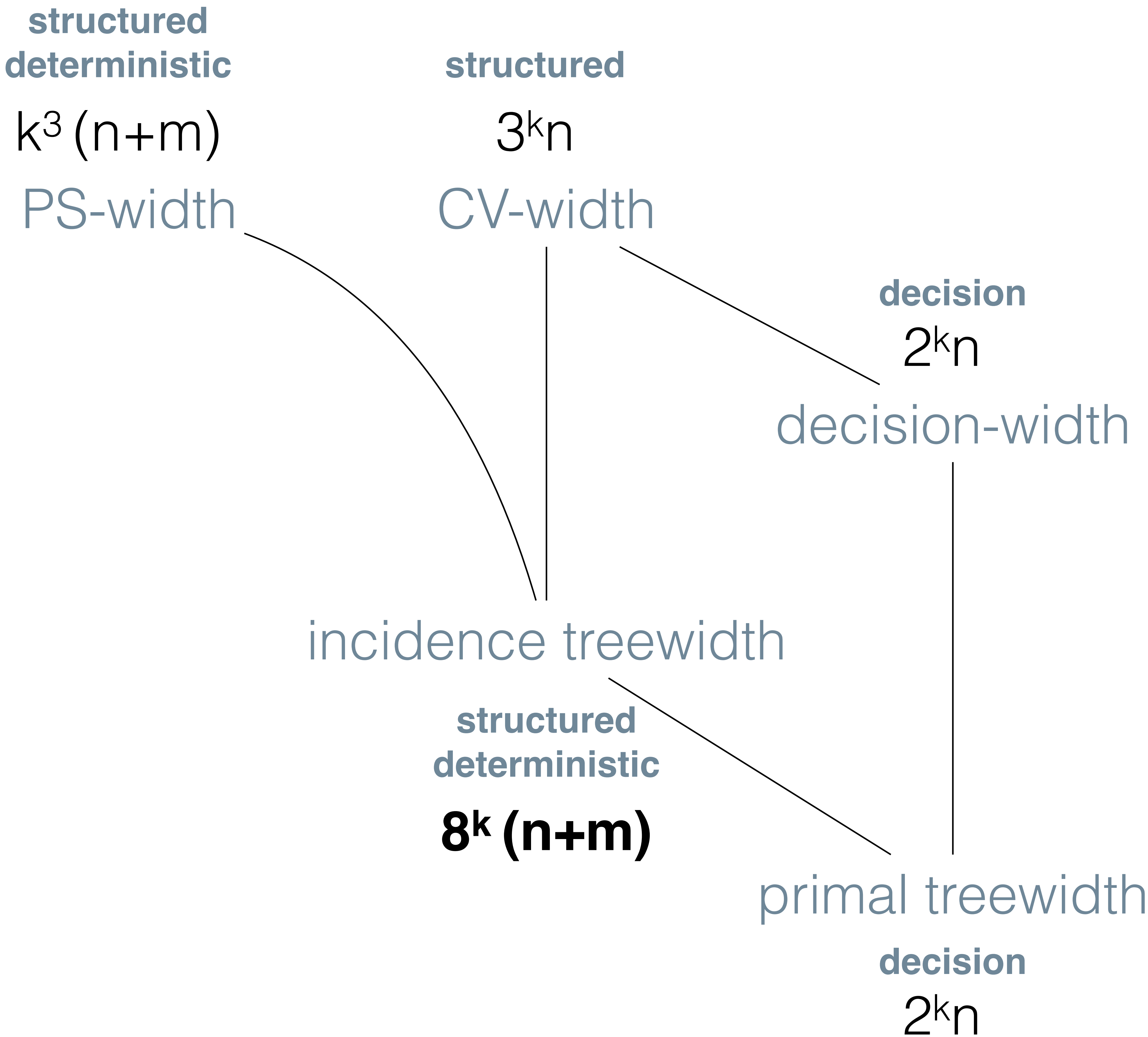
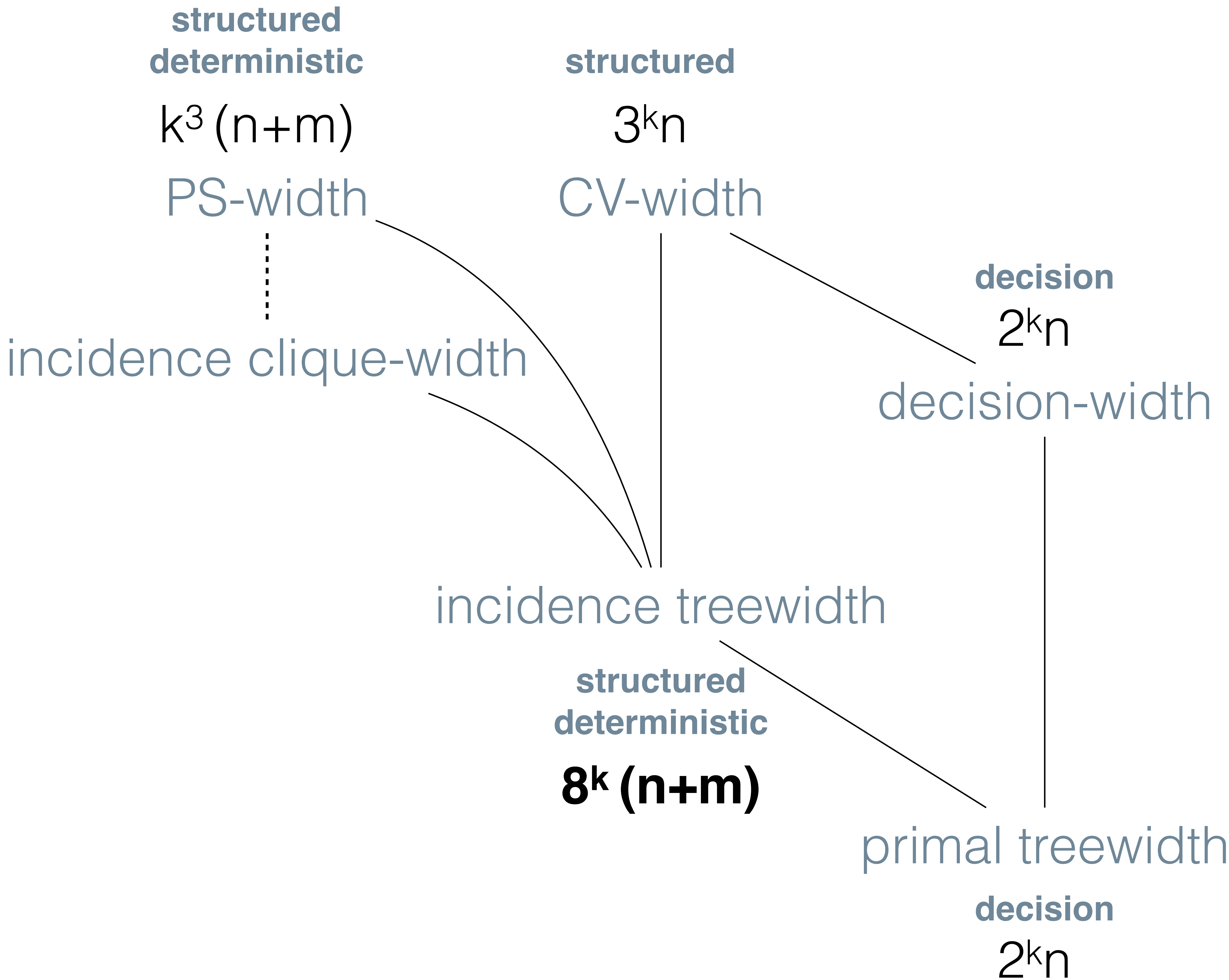

structured deterministic
$k^3 (n+m)$
PS-width
structured
$3^k n$
CV-width
decision
$2^k n$
decision-width
incidence treewidth
primal treewidth
decision
$2^k n$



structured
deterministic
k3 (n+m)
PS-width
structured
3kn
CV-width
decision
2kn
decision-width
incidence treewidth
structured
deterministic
8k (n+m)
primal treewidth
decision
2kn



structured
deterministic
k3 (n+m)
structured
3kn
PS-width
CV-width
decision
2kn
incidence clique-width
decision-width
incidence treewidth
structured
deterministic
8k (n+m)
primal treewidth
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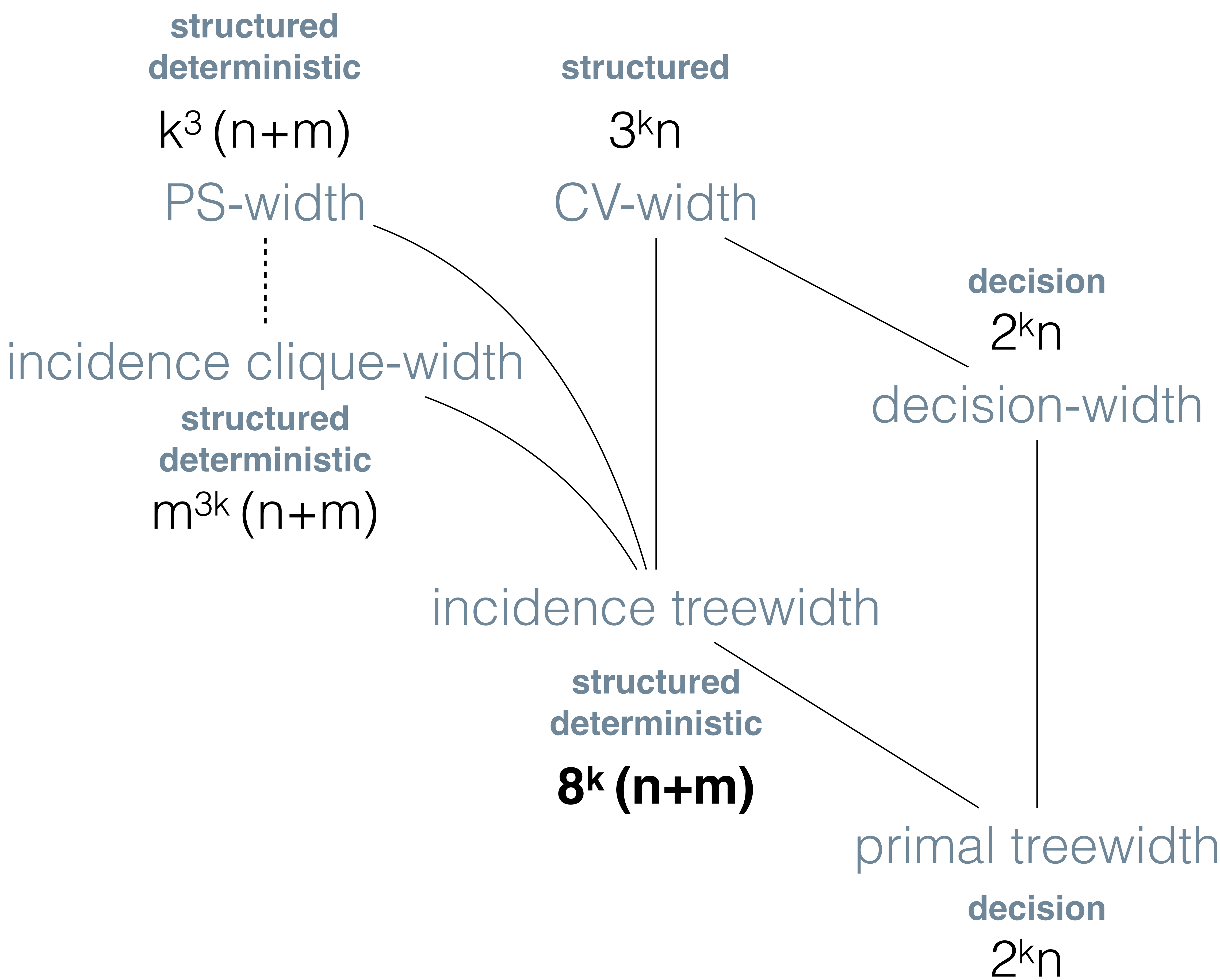
structured
deterministic
k3 (n+m)
PS-width
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3kn
CV-width
decision
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decision-width
incidence clique-width
structured
deterministic
m3k (n+m)
incidence treewidth
structured
deterministic
8k (n+m)
primal treewidth
decision
2kn

The Compilation Algorithm

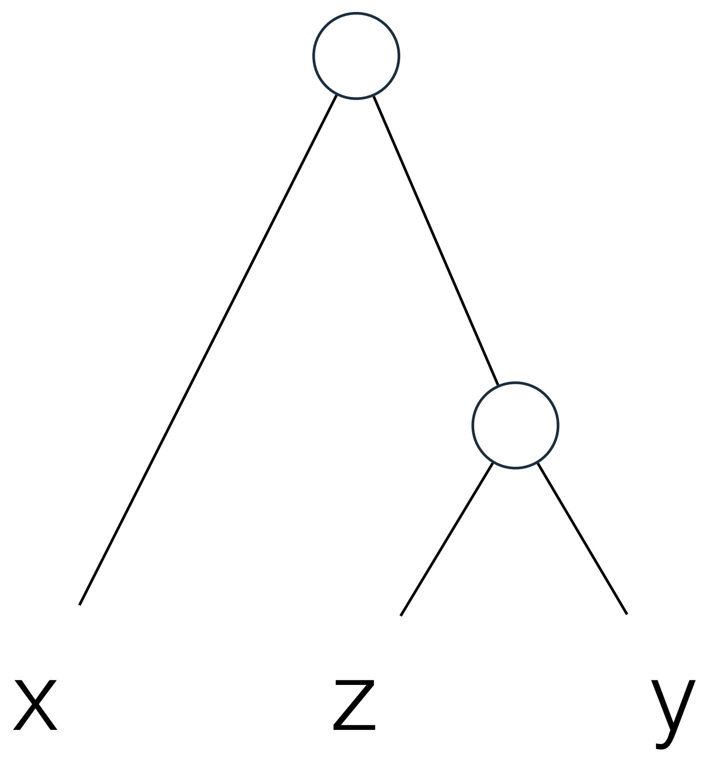
$$(x \lor \neg y \lor z) \land (\neg x \lor \neg z) \land (y \lor z)$$

$$(x \lor \neg y \lor z) \land (\neg x \lor \neg z) \land (y \lor z)$$

vtree

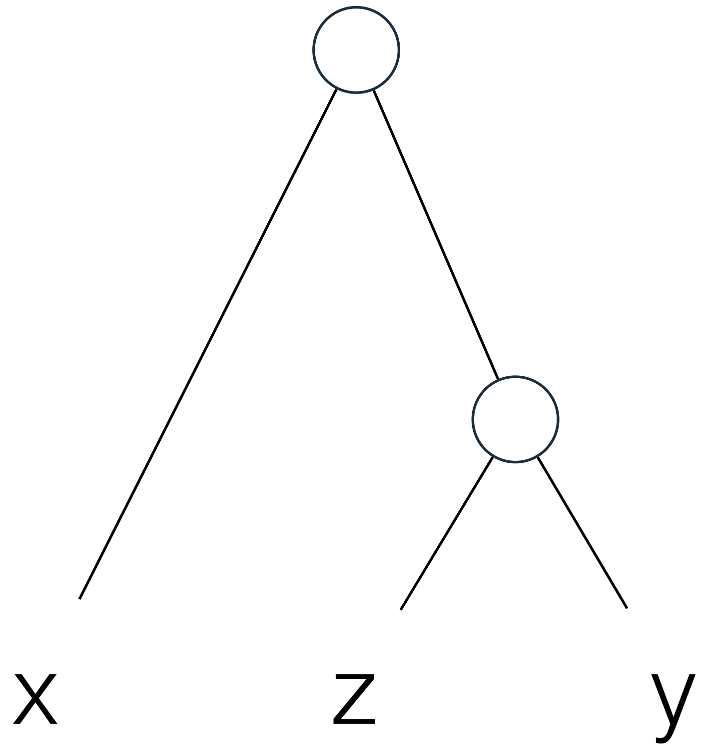
$$(x \vee \neg y \vee z) \wedge (\neg x \vee \neg z) \wedge (y \vee z)$$

vtree



$$(x \lor \neg y \lor z) \land (\neg x \lor \neg z) \land (y \lor z)$$

vtree

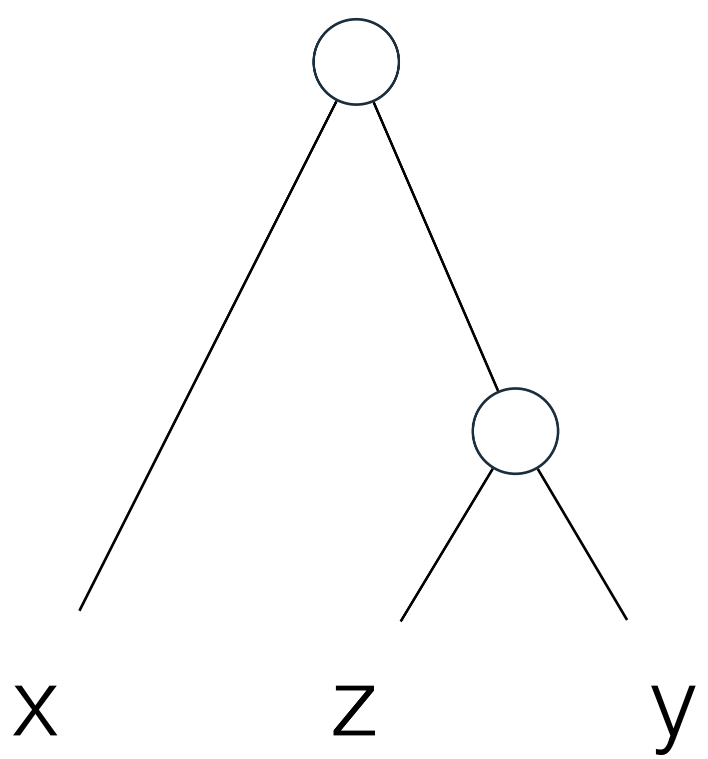


branch decomposition

C_1 C_2 C_3

$$(x \lor \neg y \lor z) \land (\neg x \lor \neg z) \land (y \lor z)$$

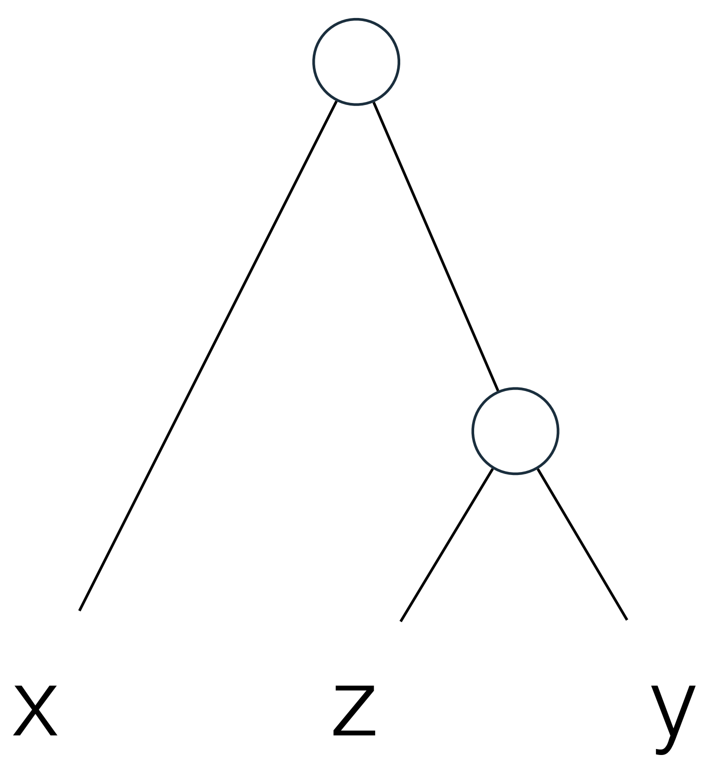
vtree



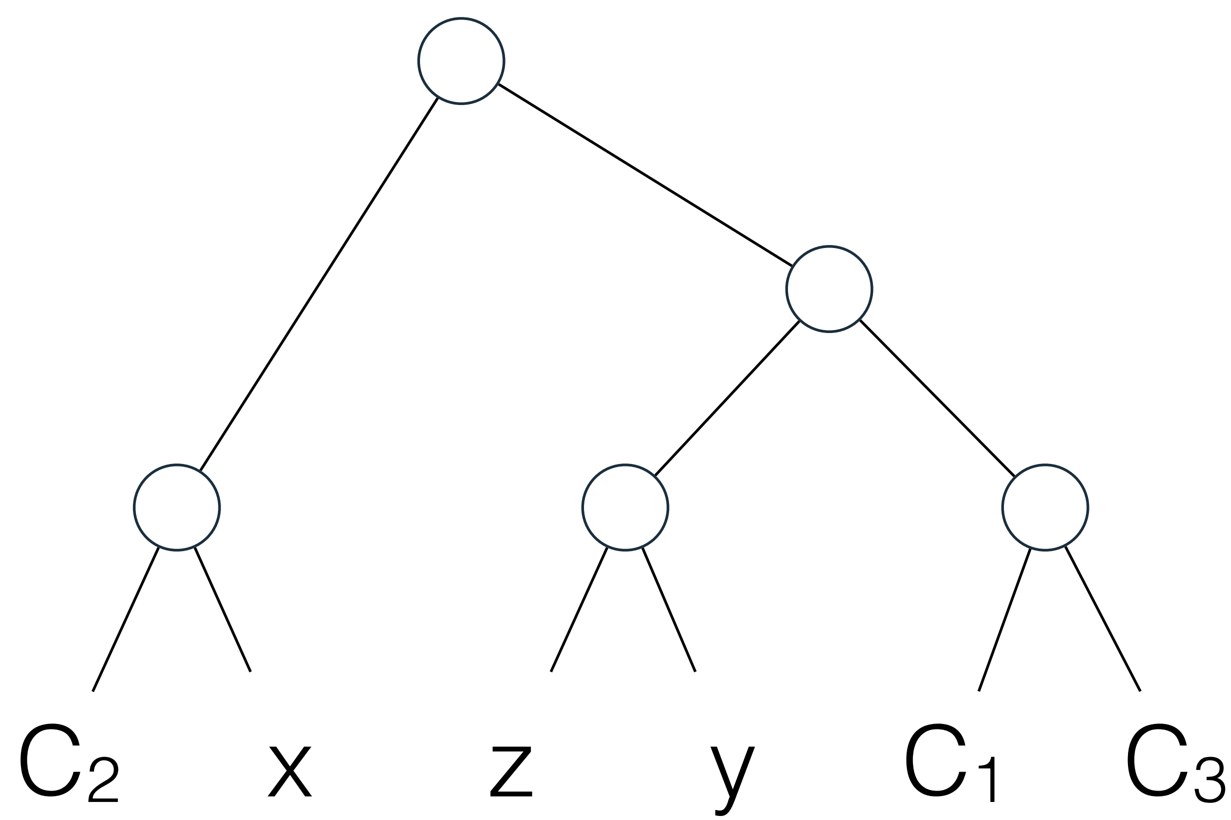
branch decomposition

$$\overset{C_1}{(x \lor \neg y \lor z)} \land \overset{C_2}{(\neg x \lor \neg z)} \land \overset{C_3}{(y \lor z)}$$

vtree



branch decomposition



Projections

The **projection** of F under assignment $\boldsymbol{\tau}$ is the set **F(τ)** of clauses of F satisfied by $\boldsymbol{\tau}$.

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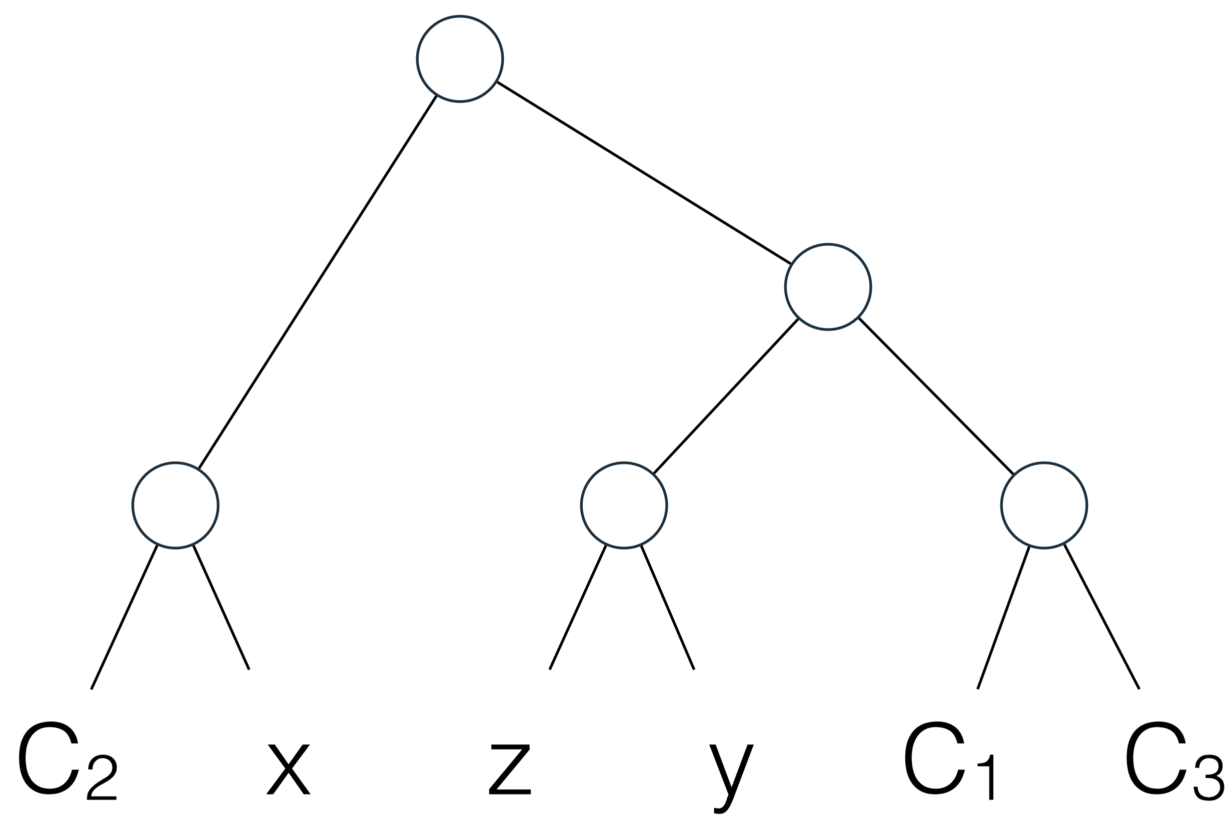
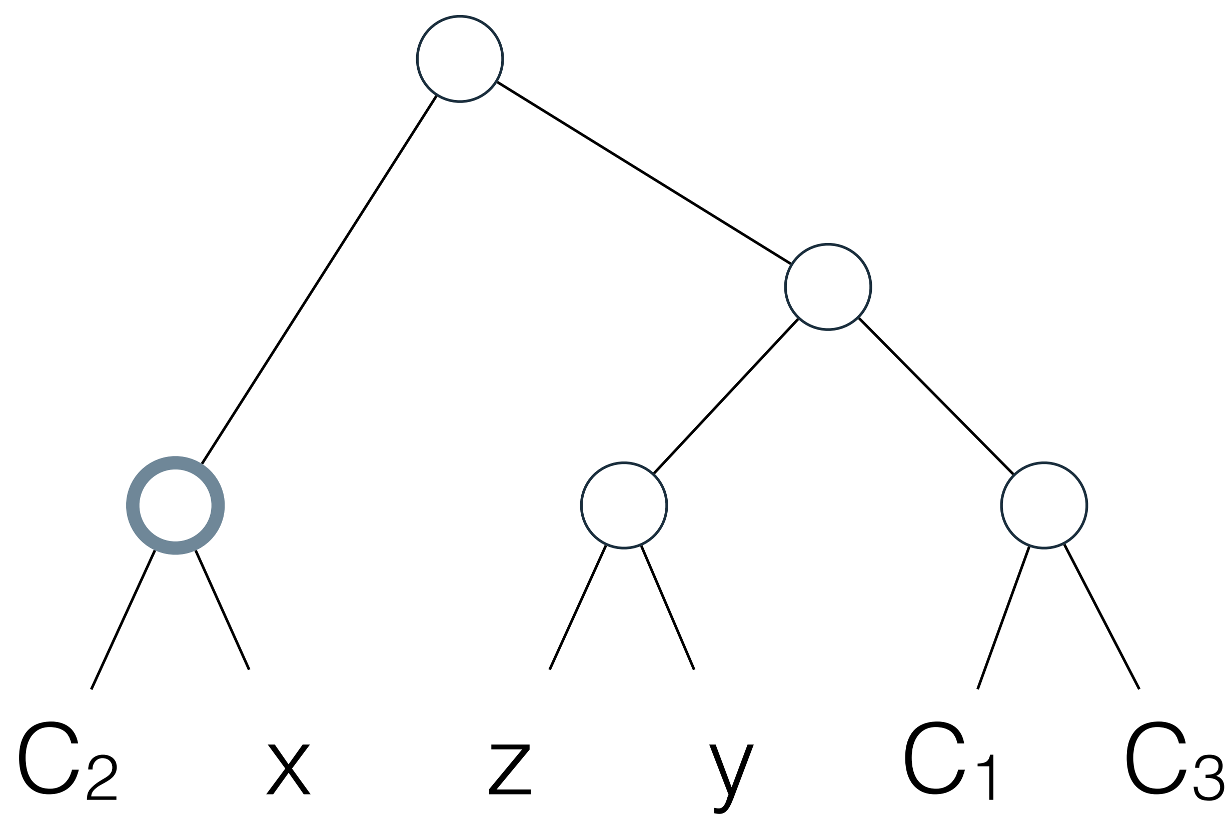
proj(F, X) the set of projections of F under assignments to X.

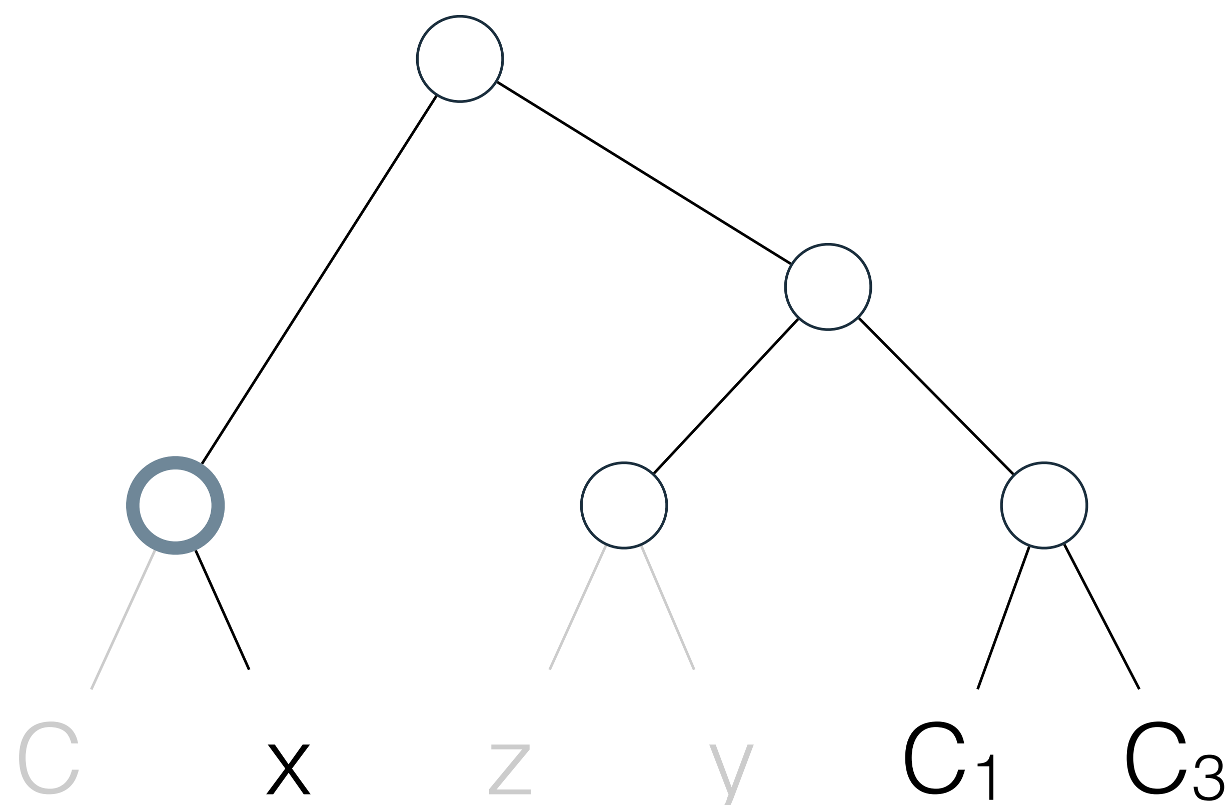
PS-width



PS-width

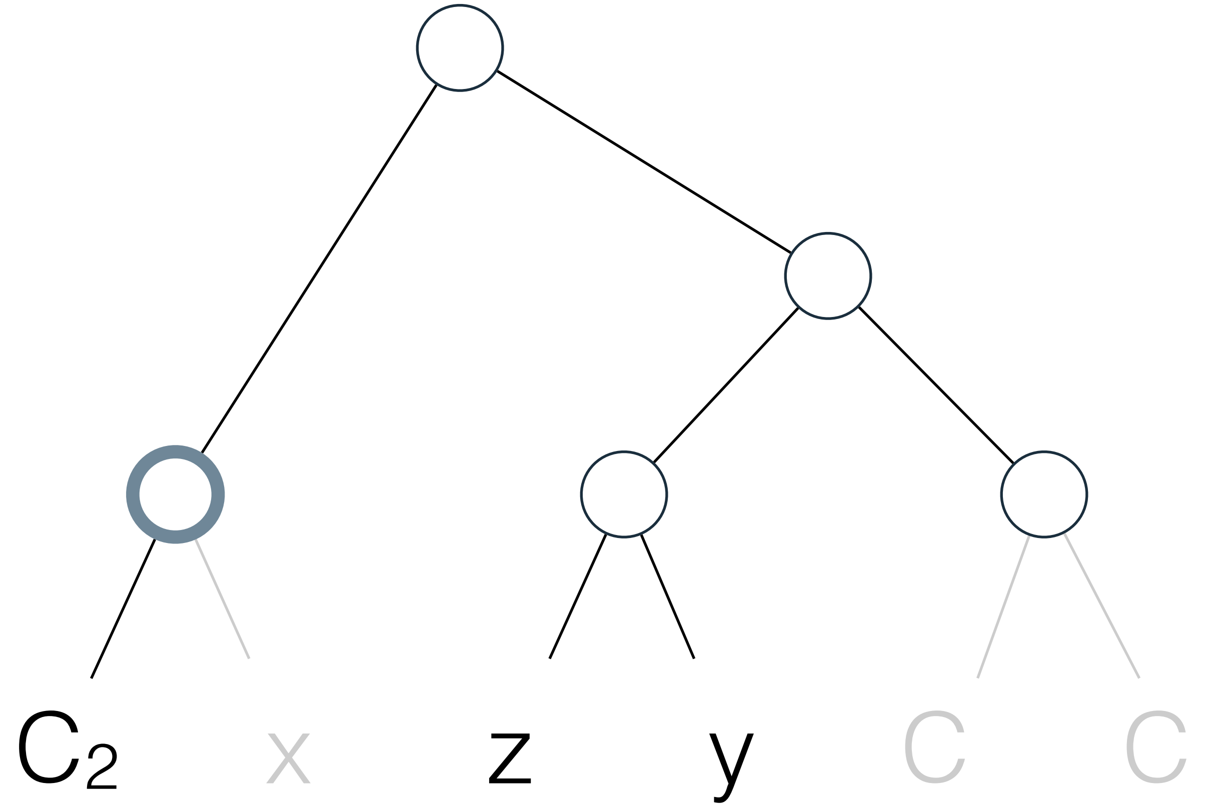


PS-width



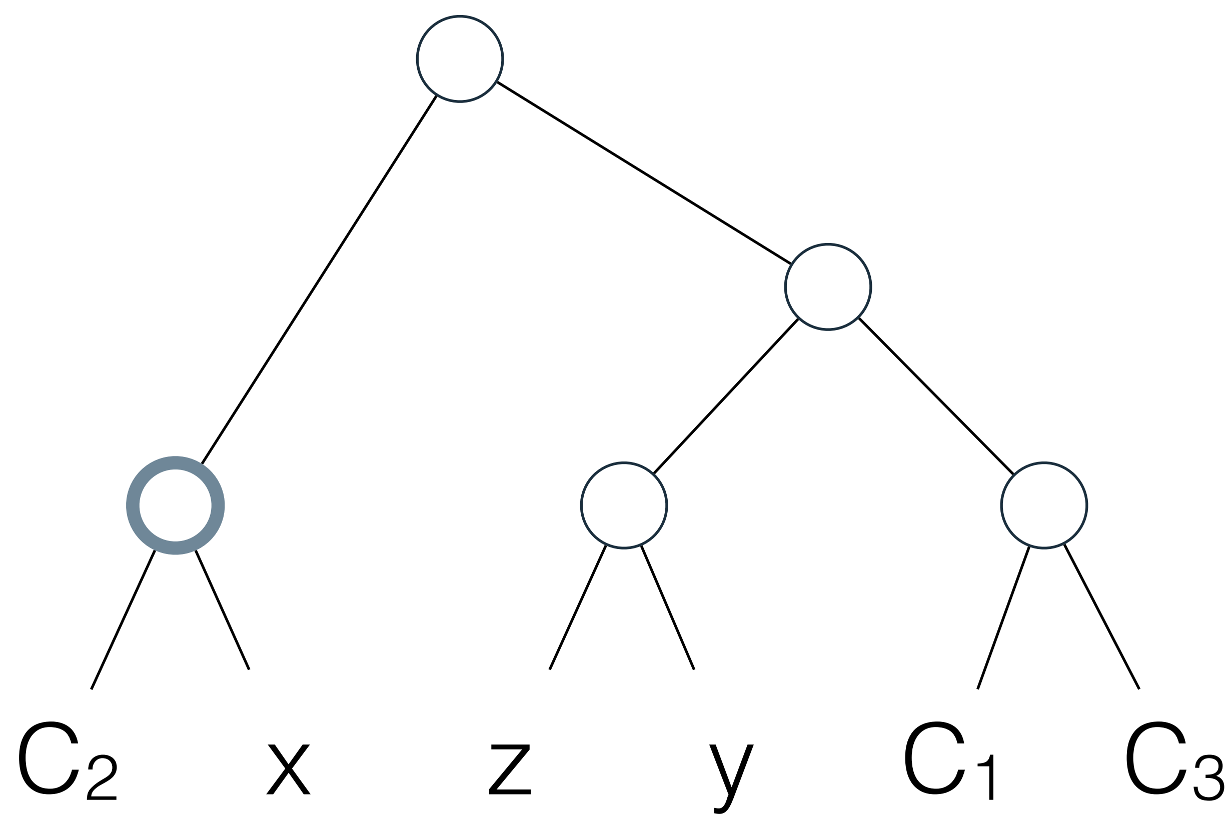
proj({C_1, C_3}, {x})

PS-width



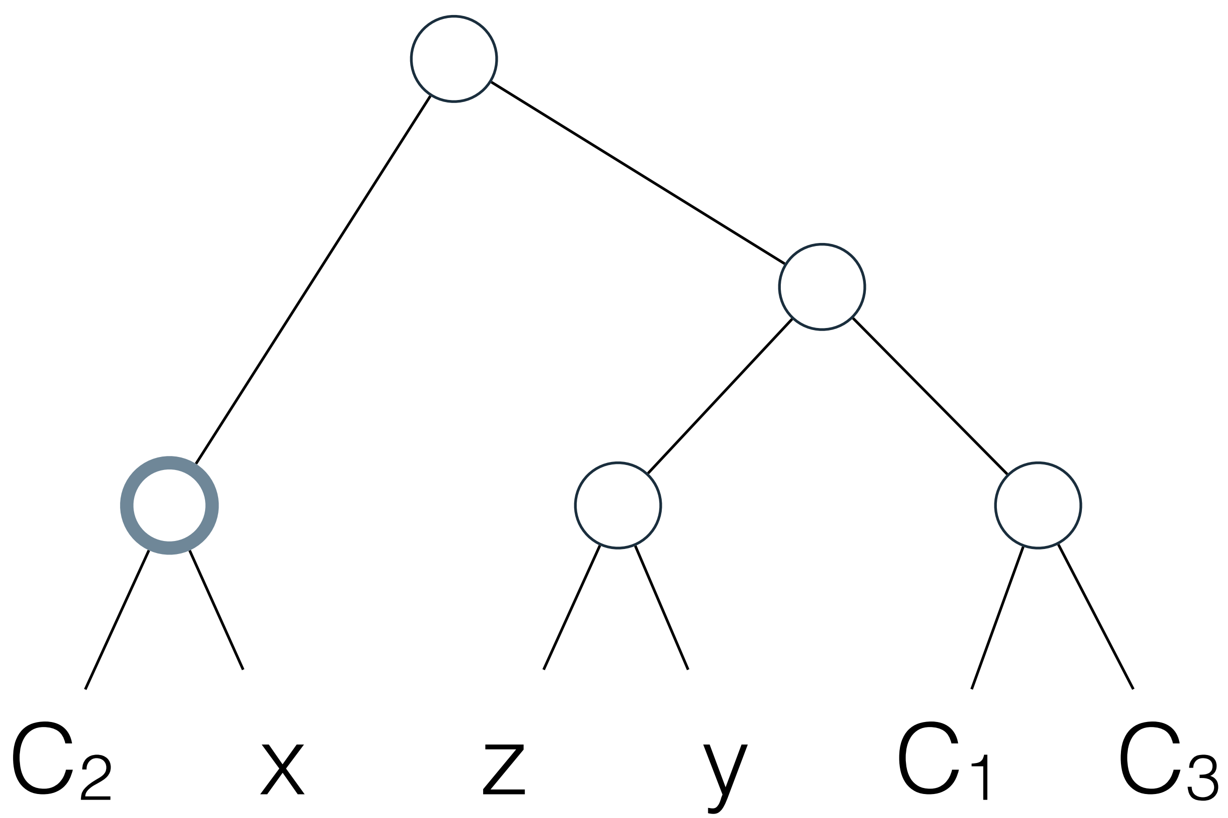
proj({C1, C3}, {x}) **proj({C2}, {z,y})**

PS-width



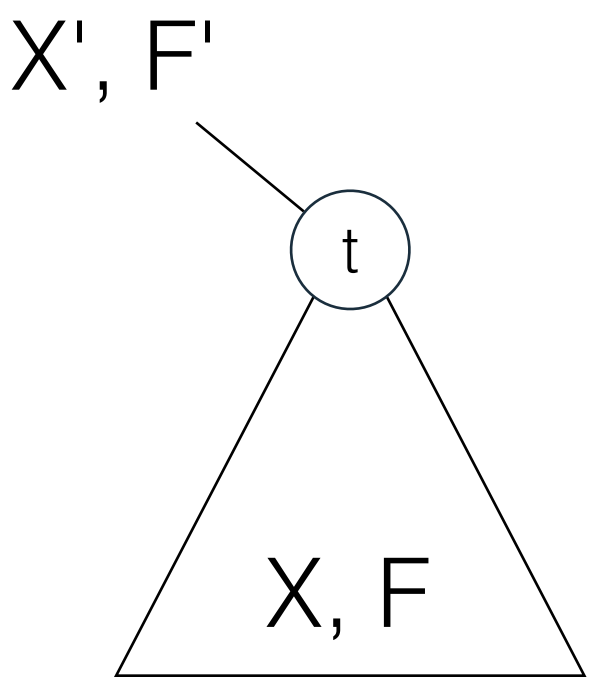
|proj({C1, C3}, {x})| |proj({C2}, {z,y})|

PS-width

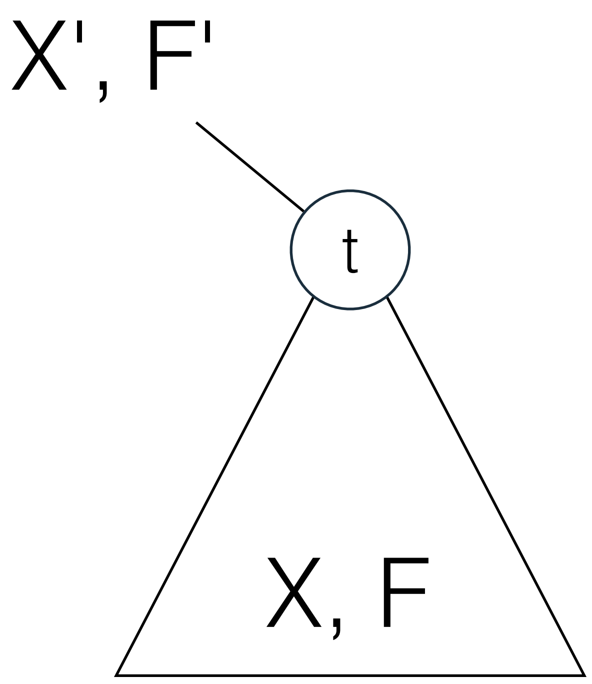


max($|proj(\{C_1, C_3\}, \{x\})|$ **,** $|proj(\{C_2\}, \{z,y\})|$ **)**

Shapes

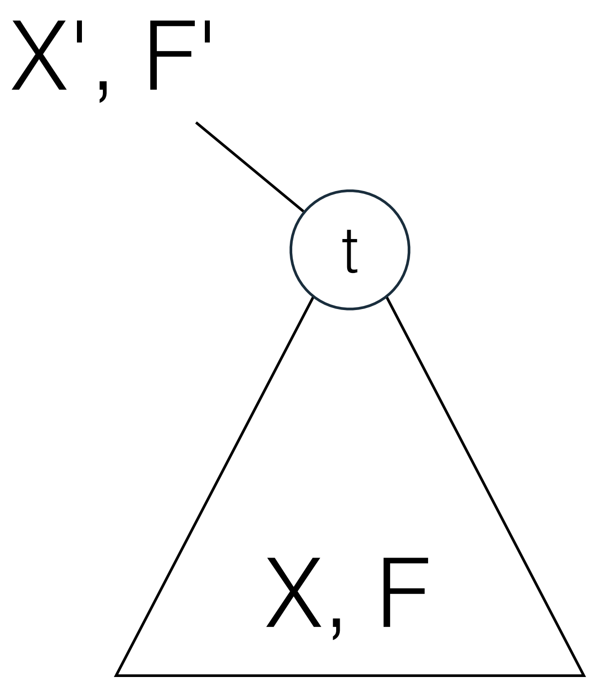


Shapes



A **shape** for t is a pair **(S, S’)** with S ∊ proj(F’, X) and S’ ∊ proj(F, X’).

Shapes



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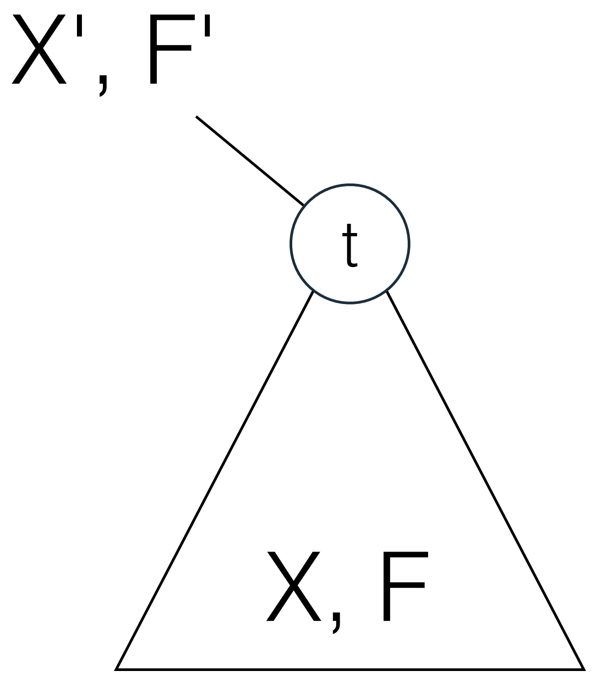
An assignment $\boldsymbol{\tau}$: X → {0,1} **has shape** (S, S') if

Shapes



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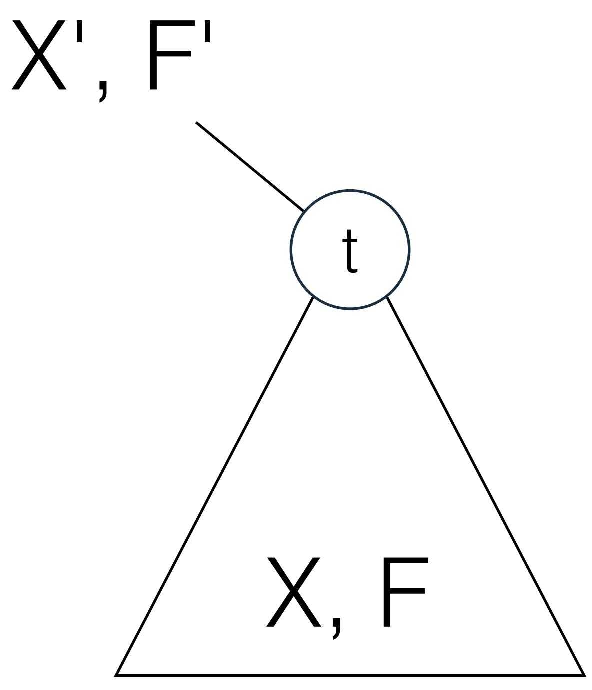
An assignment $\boldsymbol{\tau}$: X → {0,1} **has shape** (S, S') if

1. F'($\boldsymbol{\tau}$) = S

Shapes

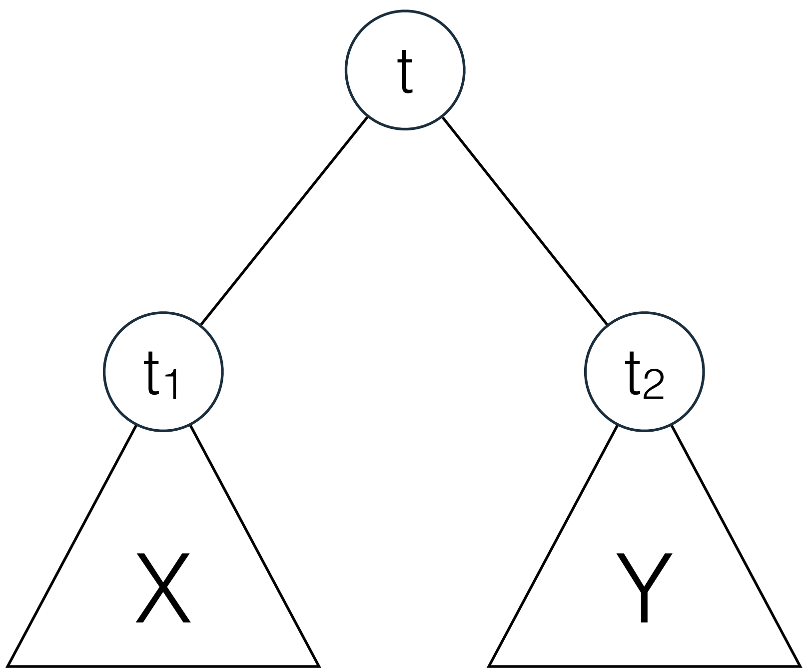


A **shape** for t is a pair **(S, S')** with $S \in \text{proj}(F', X)$ and $S' \in \text{proj}(F, X')$.

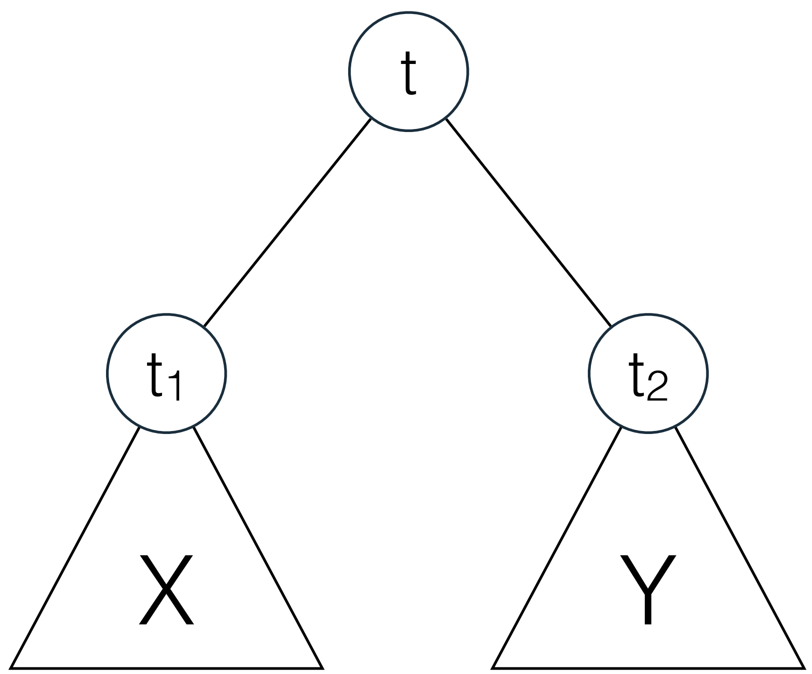
An assignment $\boldsymbol{\tau}: X \to \{0,1\}$ **has shape** (S, S') if

1. $F'(\boldsymbol{\tau}) = S$
2. $F(\boldsymbol{\tau}) \cup S' = F$

Decomposing Shapes

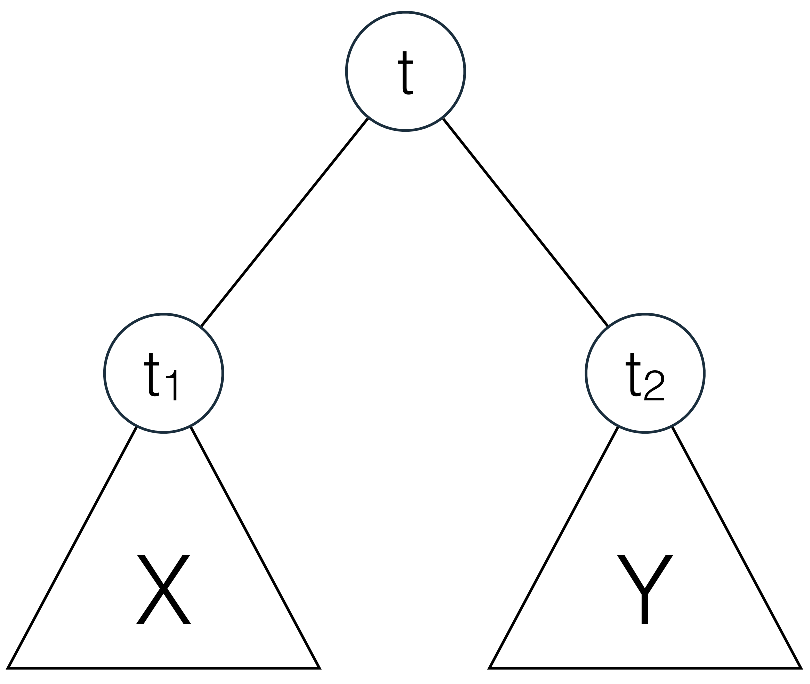


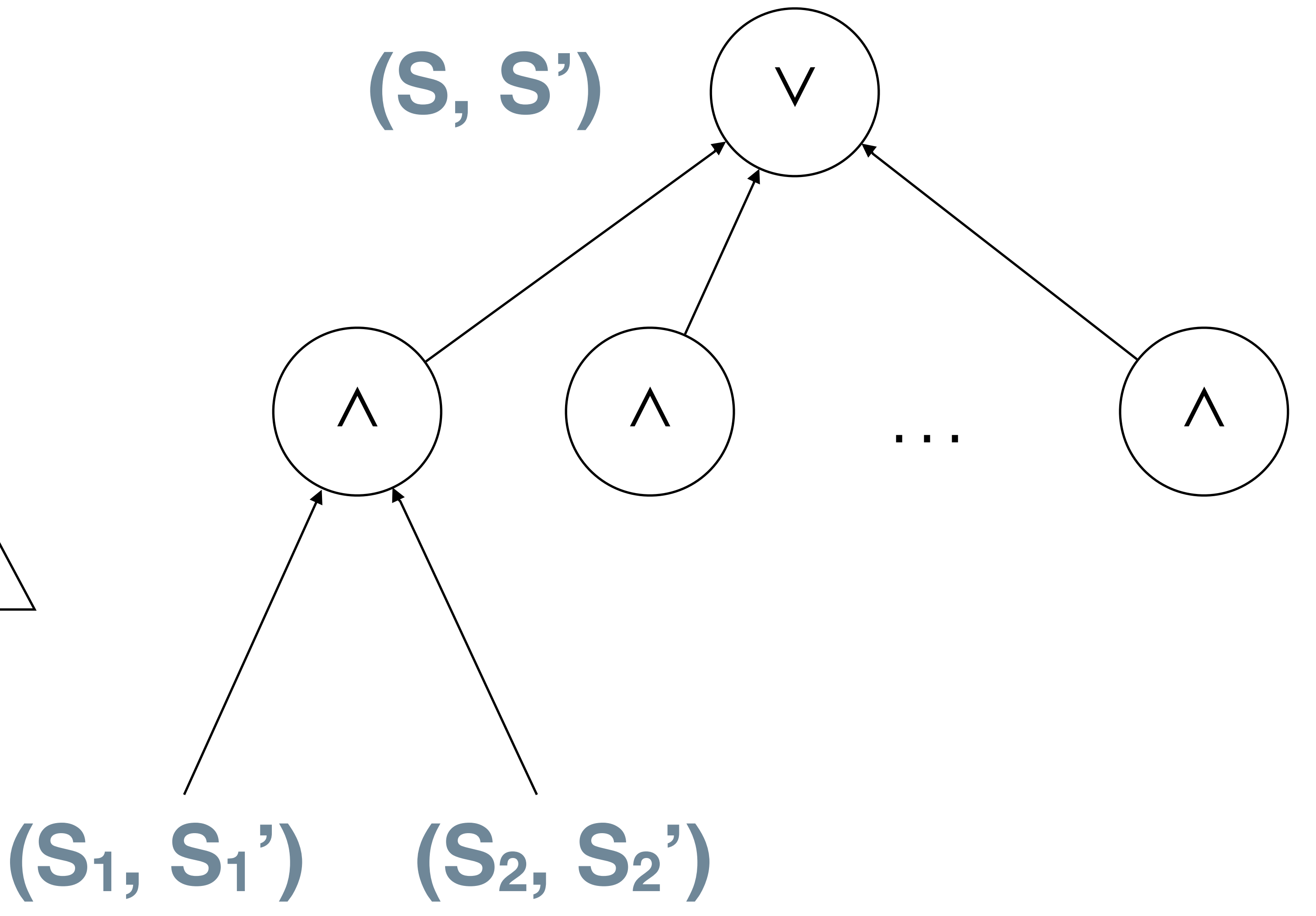
Decomposing Shapes



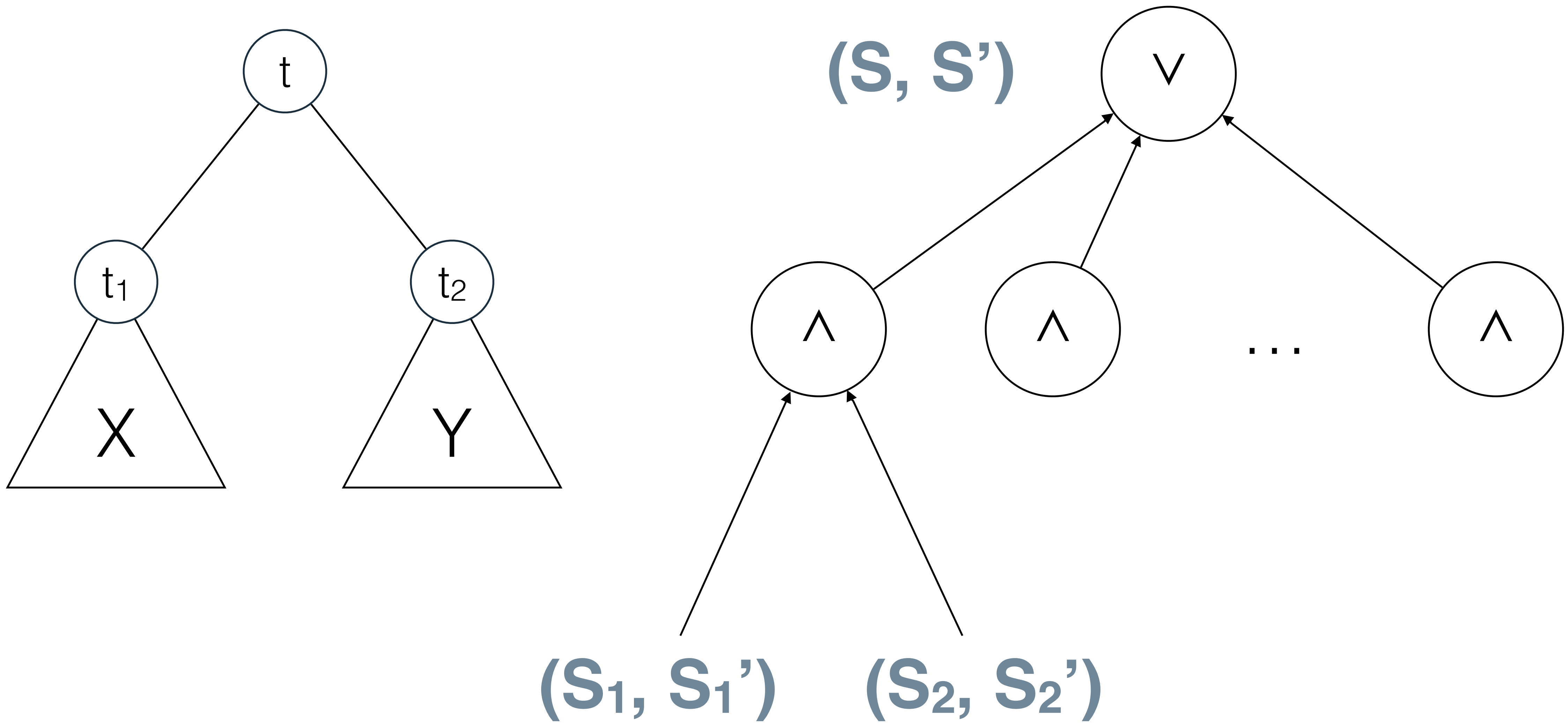
(S, S')

Decomposing Shapes



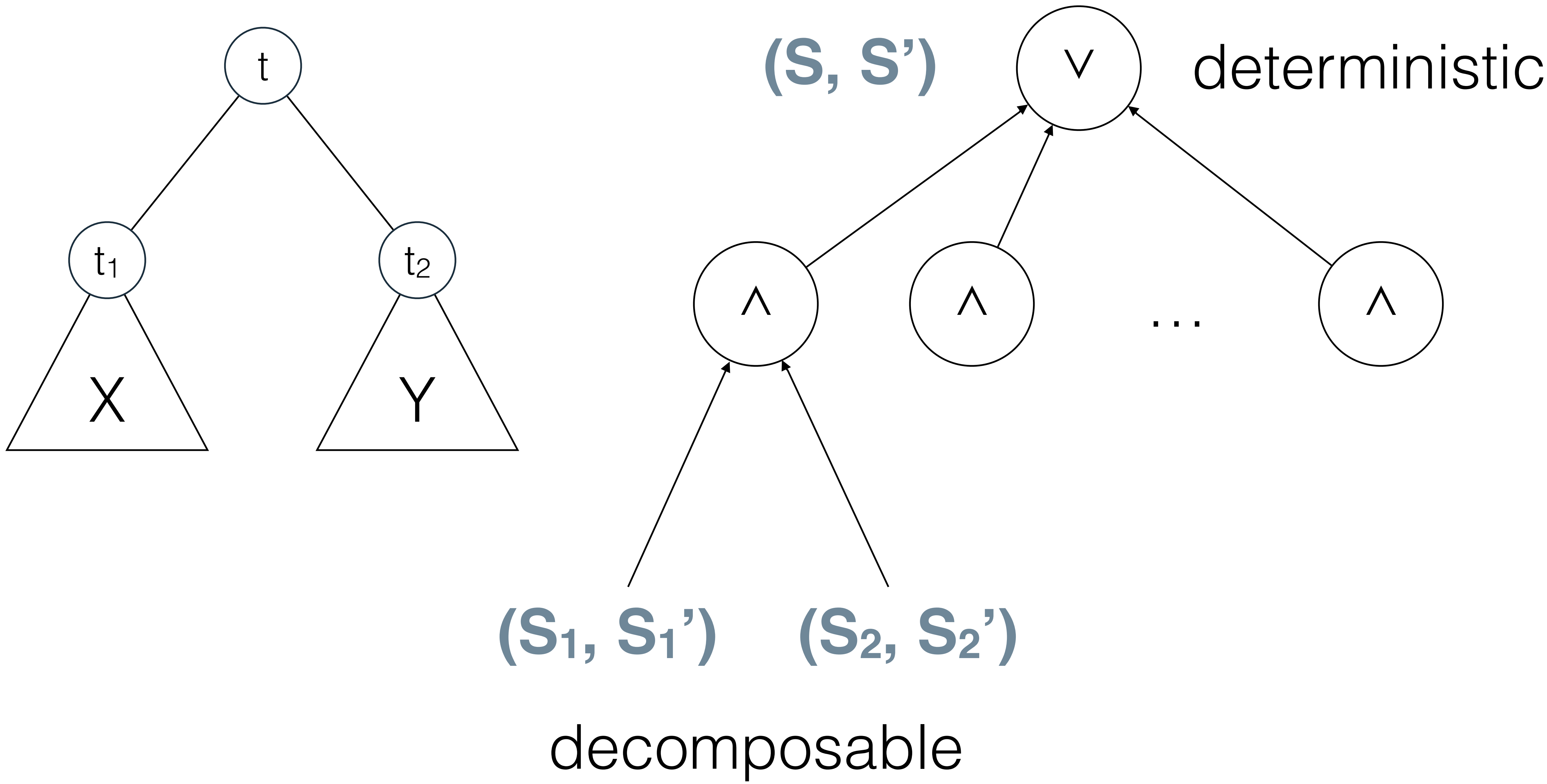


Decomposing Shapes



decomposable

Decomposing Shapes



Open Questions

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Can we compile into more restrictive languages?

decision DNNFs, SDDs

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decision DNNFs, SDDs

What is the relation between PS-width and CV-width?

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Can we compile into more restrictive languages?

decision DNNFs, SDDs

What is the relation between PS-width and CV-width?

Can decompositions of small PS-width be computed efficiently?