Parameter Compilation

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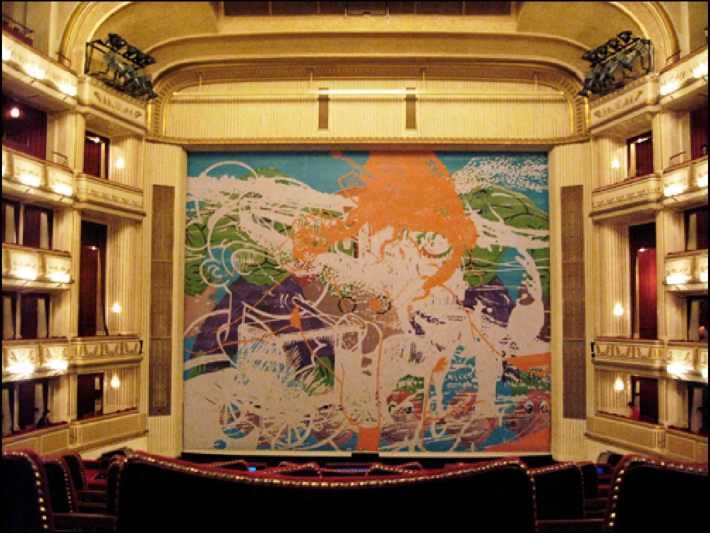

Act: Motivation

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Def: The problem $\mathrm{MC}(\Phi)$ is...

Given $\phi \in \Phi$ and a finite struct $\mathbf{B}$,
decide if $\mathbf{B} \models \phi$

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  ⇒ we might tolerate
  a non-polynomial, bad dependence on query,
  so long as have good dependence on structure
- Parameterized complexity theory: classify problems up to allowing arbitrary dependence on a parameter

  Here: the query is the parameter

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Let Φ be a set of $\{\exists, \wedge, \vee\}$-sentences, of bounded arity.

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Example of first case: define Φ to contain each $\{\exists, \wedge, \vee\}$-sentence over a unary signature;
let us use unary-EP-MC to denote $\text{MC}(\Phi)$

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- (Chen '05) "Parameterized compilability" — relax the notion of positive result; let c be "FPT-length"
- (Chen '15) "Parameter compilation" — framework for distinguishing between polynomial / non-polynomial length compilations

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Inspired by and closely related to framework
by Cadoli, Donini, Liberatore & Schaerf ’02
— see our paper for more details/discussion

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- Model checking / query evaluation:
  If a query ϕ will be posed to many databases,
  may wish to compile ϕ

The base class

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Idea: Q is decidable in polytime (in $|x|$),
modulo knowledge of $c(\kappa(x))$ — slice-wise advice

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Def: Param problem (Q, κ) *poly-comp reduces* to param problem (Q', κ') if exists:

- $g(x) = f(c(\kappa(x)), x)$ with polytime computable f, poly-length computable c
- poly-length computable $s : \Sigma^* \to \wp(\Sigma^*)$

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- $x \in Q \Leftrightarrow g(x) \in Q'$
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Prop: poly-comp-PTIME is closed under poly-comp reduction

[(Q, κ) reduces to $(Q', \kappa') \in$ poly-comp-PTIME implies $(Q, \kappa) \in$ poly-comp-PTIME]

Chopped classes

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where $g(x) = f(c(\kappa(x)), x)$ has:

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  Take $g(\phi, y) = (\phi, y)$ if y is an assignment to vars of ϕ,
  a *no* instance otherwise

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Note that the chopped classes stratify FPT...

Prop: If each lang in $\mathcal{C}$ is computable, then chopped-$\mathcal{C}$ is in FPT

Completeness

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Prop: Let $\mathcal{C}$ be a complexity class; assume that Q is $\mathcal{C}$-complete under many-one polytime reduction. Then, (Q, len) is complete for chopped-$\mathcal{C}$.

Here, len is the parameterization $\text{len}(x) = 1^n$ giving the length of a string, in unary

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- Minimal model checking is chopped-co-NP-complete

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Prop: If a param problem (Q, κ) with $Q \in \mathrm{NP}$ has a polynomial kernelization, then (Q, κ) is in chopped-NP

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- Try to classify problems of interest / established parameterized problems according to their compilability
- What can we say about color coding (embedding under bounded treewidth)?

A closing meditation

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"Kernelization is just one technique in parameterized complexity and its systematic study opened up a whole new world of research questions. Could it be that exploring other basic techniques turns out to be as fruitful as the study of kernelization?" — Dániel Marx, '12 survey