Exercise Sheet
Quantified Boolean Formulas

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1. Let A be a propositional formula. For simplicity, assume that each variable of A is among the free variables of $\exists p\ \Psi(p)$. Prove the following: $\Psi(A) \rightarrow \exists p\ \Psi(p)$.

2. Reconsider trick 1 from the lecture slides and the propositional formula Φ:
$$(A \vee \neg B \vee C) \wedge (A \vee \neg B \vee \neg D) \wedge (A \vee \neg B \vee E)$$
Show that Ψ:
$$\exists y\, (y \leftrightarrow A \vee \neg B) \wedge (y \vee C) \wedge (y \vee \neg D) \wedge (y \vee E)$$
is logically equivalent to Φ.

3. Let Ψ be the following true formula:
$$\forall x \exists y \exists z\, ((x \vee y \vee z) \wedge (x \vee \neg y \vee z) \wedge (\neg x \vee y \vee z) \wedge (x \vee z) \wedge (\neg x \vee \neg z)).$$
Provide witness formulas for the existential quantifiers and a tree-like strategy which certify that Ψ is true.

4. Translate $\forall x(\exists y(x \leftrightarrow y) \wedge \forall z(x \oplus z))$ into PCNF by prenexing and Tseitin translation. Try to keep the number of quantifier alternations low!